Theorem:- The union of an arbitrary collection of open sets is open.

Proof:- Let $\{G_i : i \in I\}$ where I is any index set be an arbitrary collection of open sets.

$G = \bigcup_{i \in I} G_i$

Consider an element $x \in G$. By definition of union x must belong to G_{i_0}, for some $i_0 \in I$

Since G_{i_0} is open, then $\exists$ a nbd 'N' of x s.t $x \in N \subseteq G_{i_0}$.

But $G_{i_0} \subseteq G$ so that $N \subseteq G$. Thus $x \in N \subseteq G$ which shows that G is open.

Theorem:- The intersection of a finite ~~funct~~ collection of open sets is open.

Proof:- Let $\{G_1, G_2, G_3, \ldots, G_n\}$ be a finite collection open sets ~~and~~.

Let $G = \bigcap_{i=1}^{n} G_i$

If $G = \phi$, then G is obviously open.
Now, we assume that G is non-empty, So Consider an element

$x \in G$

this implies that $x \in G_i$ for each $i = 1, 2, \ldots, n$

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Since each G_i is open then, there exists a nbd N_i of x such that

$$x \in N_i \subseteq G_i$$

for each $i = 1, 2, 3, \ldots, n$

$$x \in \bigcap_{i=1}^{n} N_i \subseteq \bigcap_{i=1}^{n} G_i = G$$

$$\Rightarrow x \in N \subseteq G$$

where $N = \bigcap_{i=1}^{n} N_i$

Hence intersection of finite collection of open set is open.