18/8/22

lecture-9

Product or Composition in Permutation:-

$\sigma = (1\ 2\ 3\ 4)(2\ 3\ 5)(1\ 2\ 4)(2\ 4\ 5)(1\ 4\ 5\ 3\ 2)$

$\Downarrow$

$(1\ 3\ 2\ 4)(5)$

$O(\sigma) = 4$

Working Rules:-

Step 1 Start doing calculation from right to left.

Step 2 First of all check the image of element 1 in the right most cycle and then continuing it towards leftmost cycle.

Disjoint Cycles:- Two cycles are said to be disjoint if they have no common elements when they are represented in one group.

Ex→ (i) $(1\ 2\ 3\ 4)(5)$

(ii) $(1\ 2\ 3\ 4)(5\ 6\ 7)$

$(1\ 2\ 3\ 4)(3\ 4\ 5)$ This is not disjoint cycle.

Theorem (1): "Every permutation can be written as product of disjoint cycles."

Ex → $(1\ 2\ 3\ 4)(3\ 4\ 5) = (1\ 2\ 3)(4\ 5)$

Theorem (2) Let σ be any permutation such that $\sigma = \sigma_1 \sigma_2 \sigma_3 \ldots \sigma_k$ where σ_j, $j = 1, 2, 3, \ldots k$ where σ_j are disjoint cycles then

$\boxed{\text{Order of } \sigma = O(\sigma) = \text{lcm}\{O(\sigma_1)\, O(\sigma_2) \ldots O(\sigma_k)\}}$

Ex → (i) $\underbrace{(1\ 2\ 3\ 4)}_{\sigma_1}\underbrace{(5\ 6\ 7)}_{\sigma_2}$

$O(\sigma_1) = 4$

$O(\sigma_2) = 3$

$O(\sigma) = \text{lcm}\{4, 3\} = 12$ Ans

(ii) $\underbrace{(1\ 2\ 3)}_{\sigma_1}\underbrace{(4\ 5)}_{\sigma_2}$

$O(\sigma_1) = 3$

$O(\sigma_2) = 2$

$O(\sigma) = \text{lcm}\{3, 2\} = 6$ Ans

Q. (i) Find order of $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 1 & 5 & 6 & 8 & 9 & 7 \end{pmatrix}$

$$\sigma = \underbrace{(1\ 2\ \overset{4}{3}\ 4)}_{\sigma_1}\ \underbrace{(\overset{1}{5})}_{\sigma_2}\ \underbrace{(\overset{1}{6})}_{\sigma_3}\ \underbrace{(7\ \overset{3}{8}\ 9)}_{\sigma_4}$$

$O(\sigma) = \text{lcm}\{4, 1, 1, 3\}$

$= 12$ Ans

(ii) $\sigma^{24} = ?$

$\sigma^{24} = (\sigma^{12})^2$

$= (I)^2$

$\sigma^{24} = I$ Ans

(iii) $\sigma^{25} = \sigma^{24} \cdot \sigma$

$= I \cdot \sigma = \sigma$ Ans

(iv) $\sigma^{38} = (\sigma^{12})^3 \cdot \sigma^2 = I \cdot \sigma^2 = \sigma^2$ Ans

(v) $\sigma^{2022} = (\sigma^{12})^{16} \sigma^6$

$= I \cdot \sigma^6$

$= \sigma^6$ Ans

Inverse of a Permutation :-

$\sigma = (a\ b\ c\ d)$
$\sigma^{-1} = (d\ c\ b\ a)$

$(1\ 4\ 2)^{-1} = (1\ 2\ 4)$

$(2\ 5\ 7)^{-1} = (2\ 7\ 5)$

$(1\ 2\ 3\ 4)^{-1} = (4\ 3\ 2\ 1)$
$= (1\ 4\ 3\ 2)$

H.W. Exercise →

① Verify → $\sigma\sigma^{-1} = I$ where $\sigma = (a\ b\ c\ d)$

② $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 2 & 5 & 4 \end{bmatrix}_{2\times5}$

Given matrix A is permutation matrix defined by σ then which of the following is/are correct?

ⓐ $A = A^{-2}$
ⓑ $A = A^{-1}$
ⓒ $A = A^{-5}$
ⓓ $A = A^{-11}$
ⓔ None of these