(ii) Order Axioms :- The order relation $>$ called greater than between pairs of real numbers satisfy the following axioms —

(O1) Law of Trichotomy :- If $a, b \in R$, then one and only one of the following is true :—

- $a > b$ • $a < b$ • $a = b$

(O2) Transitivity law :- If $a, b, c \in R$, then —
$$a > b,\ b > c \Rightarrow a > c$$

(O3) Monotone Property for Addition :- If $a, b, c \in R$ then —
$$a > b \Rightarrow a + c > b + c$$

(O4) Monotone Property for Multiplication :-

If $a, b \in R$ and $c > 0$ then —
$$a > b \Rightarrow ac > bc$$

In view of the above axioms, the set R is said to be ordered.
Then R is an ordered field.

R^+ is the set of positive real numbers
R^- is the set of negative real numbers

(iii) Completeness Axiom:- Every non-empty bounded above subset of real numbers R has a supremum in R.

Properties of Real Numbers:-

→ Algebraic Properties →

① The additive identity in R is unique.

② The additive inverse of each element is unique.

③ If $a, b, c \in R$, then
$a+c = b+c \Rightarrow a=b$; $c+a = c+b \Rightarrow a=b$

④ $a+b=0 \Rightarrow b=-a$

⑤ $-(-a) = a$

⑥ The multiplicative identity in R is unique.

⑦ The multiplicative inverse of each element in R is unique.

⑧ If $c \neq 0$, then $a \cdot c = b \cdot c \Rightarrow a=b$ &
$c \cdot a = c \cdot b \Rightarrow a=b$

(9) $a \cdot b = 1 \implies b = a^{-1}$

(10) If $a \neq 0$, then $(a^{-1})^{-1} = a$

(11) $a \cdot 0 = 0 \quad \forall \; a \in R$

(12) (i) $a \neq 0$ and $b \neq 0 \implies ab \neq 0$

(ii) $a \cdot b = 0 \iff$ either $a = 0$ or $b = 0$

(13) $a \cdot (-b) = -(a \cdot b) = (-a) \cdot b$

(14) $(-a)(-b) = a \cdot b$, $(-1) \cdot a = -a$

(15) $(a \cdot b)^{-1} = a^{-1} b^{-1}$ provided $a \neq 0, b \neq 0$

(16) If $a, b \in R$ then the eq^n $x + a = b$ has a unique sol^n $x = b - a$ in R.

(17) If $a, b \in R$ and $a \neq 0$ then the eq^n $a \cdot x = b$ has a unique sol^n $x = \frac{b}{a}$ in R.