Order Properties of Real Numbers:-

(1) For each real number 'a' one and only one of the following holds —
$a > 0, \quad a = 0, \quad -a > 0$

(2) $a < 0$ iff $-a > 0$

(3) $a > b$ iff $a - b > 0$

(4) If $c < 0$, $a > b \Rightarrow ac < bc$

(5) If $a, b \in \mathbb{R}^+$ and $a > b \Rightarrow \frac{1}{a} < \frac{1}{b}$

Extended Real Number System:-

The symbols ∞ and $-\infty$ are extremely useful even though these are not real numbers. It is often convenient to extend the system of real numbers by adjoining ∞ and $-\infty$. The enlarged set is called the ~~ex~~ extended real number system.

If 'a' is any real number then —

- $-\infty < a < \infty$
- $a + \infty = \infty = \infty + a$; $a - \infty = -\infty = -\infty + a$
- $\frac{a}{\infty} = 0$; $\frac{\infty}{a} = \infty \times a = a \times \infty = \begin{cases} \infty & \text{if } a > 0 \\ -\infty & \text{if } a < 0 \end{cases}$
- $\infty \times \infty = (-\infty) \times (-\infty) = \infty + \infty$
- $\infty \times (-\infty) = (-\infty) \times \infty = -\infty - \infty$

The following combination remain meaningless
$\infty-\infty$, $-\infty+\infty$, $0\times\infty$, $\infty\times 0$, $\frac{\infty}{\infty}$

Intervals :-

If $a,b\in R$ and $a<b$ we define (i) open interval →
It is denoted by (a,b) and is defined as the set $(a,b)=\{x\in R: a<x<b\}$
The points a & b are called end points of (a,b)

(ii) Closed Interval :- It is denoted by $[a,b]$ and is defined as the set $[a,b]=\{x\in R: a\leq x\leq b\}$
The points a & b are called end points of closed interval $[a,b]$.

(iii) Semi-open or semi-closed interval →
$[a,b)=\{x\in R; a\leq x<b\}$
$(a,b]=\{x\in R: a<x\leq b\}$

(iv) Infinite open interval →
$(a,\infty)=\{x\in R: x>a\}$
$(-\infty,a)=\{x\in R: x<a\}$

(v) Infinite closed interval →
$[a,\infty)=\{x\in R: x\geq a\}$
$(-\infty,a]=\{x\in R: x\leq a\}$