## Absolute Value (or Modulus) of a real numbers

The absolute value (or modulus) of a real number '$x$' denoted by $|x|$ and defined by —

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

## Properties of absolute value of real nos:-

If $x, y \in R \Longrightarrow$

1. $|x| \geq 0$ ; $|x|^2 = x^2$
2. $|-x| = |x|$
3. $|x| = \max\{x, -x\}$
4. $-|x| = \min\{x, -x\}$
5. $|xy| = |x||y|$
6. $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$ provided $y \neq 0$
7. $|x+y| \leq |x| + |y|$
8. $|x-y| \leq |x| + |y|$
9. $|x-y| \geq |x| - |y|$
10. $|x-y|$
11. $|x-y| \geq |(|x| - |y|)|$
12. $|x| < \epsilon \Longleftrightarrow -\epsilon < x < \epsilon$

    $\Longleftrightarrow x \in (-\epsilon, \epsilon)$
13. $|x-a| < \epsilon \Longleftrightarrow a-\epsilon < x < a+\epsilon$

    $\Longleftrightarrow x \in (a-\epsilon, a+\epsilon)$
14. If $x = y \Longrightarrow |x| = |y|$

    but converse need not be true.

    Ex→ $|-3| = |3|$ but $-3 \neq 3$

## Bounded and Unbounded Sets:-

A subset 'S' of R is said to be bounded above if $\exists$ a real number 'K' such that every element of S is less than or equal to K i.e.

$$\boxed{x \leq K \ \forall \ x \in S}$$

The number 'K' in this case is called an upper bound of S.

A subset 'S' of R is said to be bounded below if $\exists$ a real number 'k' such that every element of S is greater than or equal to k i.e.

$$\boxed{x \geq k \ \forall \ x \in S}$$

The number 'k' in this case is called a lower bound of S.

A subset S of R is said to be bounded if it is bounded above as well as bounded below. Thus, S is bounded if $\exists$ real numbers k & K s.t. —

$$\boxed{k \leq x \leq K \ \forall \ x \in S}$$

Equivalently, a subset S of R is bounded if $\exists$ a real number $K \geq 0$ s.t. —

$$\boxed{|x| \leq K \ \forall \ x \in S}$$

A subset S of R is said to be unbounded if it is not bounded.