Ex ① The set $N$ of natural numbers is bounded below by 1 but unbounded above.

② The set of negative integers is bounded above by 0 but unbounded below.

③ Intervals $[a,b]$, $(a,b)$, $[a,b)$, $(a,b]$ are bounded below by 'a' and bounded above by 'b'

④ The set $\left\{\frac{1}{n} : n \in N\right\}$ is bounded

⑤ The set $Q$, $I$ and $R$ are unbounded.

⑥ Every finite set is bounded.

Maximum (or largest element) of a set :-

A non-empty subset $S$ of $R$ contains a largest element $M$ i.e. $x \le M \ \forall \ x \in S$ where $M$ is called the maximum (or largest) element of set $S$.

Minimum (or smallest element) of a set :-

A non-empty subset $S$ of $R$ contains a smallest element $m$ i.e. $x \ge m \ \forall \ x \in S$ then 'm' is called the minimum (or smallest) element of set $S$.

Ex $S = \{1, 2, 3, 4\}$
Max $S = 4$
Min $S = 1$

Ex $S = \left\{1, \frac{1}{2}, \frac{1}{3}, \ldots\right\}$
Max $S = 1$, while $S$ does not contain the minimum element.

## Supremum (or least Upper Bound) (Sup or lub)

If $\alpha$ is an upper bound of a subset $S$ of $\mathbb{R}$ and any real number less than $\alpha$ is not an upper bound of $S$, then $\alpha$ is called Supremum (or least upper bound) of $S$. In this case, we write —

$$\text{Sup } S = \alpha \quad \text{or} \quad \text{lub } S = \alpha$$

Thus a real number $\alpha$ is supremum of $S$ if —
(i) $\alpha$ is an upper bound of $S$.
(ii) $\alpha \le K$ for every upper bound '$K$' of $S$.