Infimum (or greatest lower bound):

If β is a lower bound of a subset S of R any real number greater than β is not a lower bound of S, then β is called infimum (or glb) of S.
In this case, we can write:

$$\text{Inf } S = \beta \quad \text{or} \quad \text{glb } S = \beta$$

Thus a real number β is infimum of S if —
(i) β is a lower bound of S.
(ii) $\beta \geq K$, for every lower bound 'K' of S.

Ex ① $S = \left\{ \frac{1}{n} : n \in \mathbb{N} \right\}$
Sup $S = 1$
Inf $S = 0$

② $S = [2, 3]$
Sup $S = 3$
Inf $S = 2$

③ $S = (2, 3)$
Sup $S = 3$ & Inf $S = 2$

④ S = set of natural numbers
Sup S = does not exists
Inf $S = 1$

⑤ The sets Z, Q, I and R have neither supremum nor Infimum.

⑥ $S = \{1, 3, 5, \ldots, 2n+1\}$ be a finite set

$\text{Sup } S = 2n+1$

$\text{Inf } S = 1$

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Some Properties of Supremum & Infimum:

① If $\text{Sup } S = \alpha$ and $\text{Inf } S = \beta$ then $\beta \leq \alpha$

② If K is an upper bound of S and $K \in S$ then $\text{Sup } S = K$

③ If k is a lower bound of S and $k \in S$ then $\text{Inf } S = k$

④ If max S exists, then $\text{Sup } S = \max S$

⑤ If min S exists, then $\text{Inf } S = \min S$

⑥ If A and B are non-empty subsets of $\mathbb{R}$, then

$$\text{Inf}(A \cup B) = \min\{\text{Inf } A, \text{Inf } B\}$$

$$\text{Sup}(A \cup B) = \max\{\text{Sup } A, \text{Sup } B\}$$

If A and B are non-empty subsets of $\mathbb{R}$ such that $A \subset B$, then —

$$\text{Inf } B \leq \text{Inf } A \leq \text{Sup } A \leq \text{Sup } B$$