Plasma → The word plasma is used to describe a wide variety of macroscopically neutral substances containing many interacting free e^- and ionized atoms or molecules, which exhibit collective behaviour due to the long-range coulomb forces.

→ For a collection of interacting charged and neutral particles to exhibit plasma behaviour it must satisfy certain conditions or criteria for plasma existence.

Plasma as the fourth state of matter →

four states of matter

→ Liquid
→ Gaseous
→ Solid
→ Plasma

The Basic Eqns. of Magnetohydrodynamics ①

The behaviour of a continuous plasma is governed by a simplified form of Maxwell's equations with Ohm's law, a gas law and equations of mass continuity, motion and energy.

Now consider Maxwell's equations in M.K.S units,

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} \quad -①$$

$$\nabla \cdot \vec{B} = 0 \quad -②$$

$$\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t} \quad -③$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad -④$$

$$\vec{H} = \frac{\vec{B}}{\mu}, \quad \vec{D} = \epsilon \vec{E}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \simeq 2.998 \times 10^{8} \ m s^{-1}$$

where $4\pi \times 10^{-7}$ henry $m^{-1} = \mu_0$

8.854×10^{-12} farad $m^{-1} = \epsilon_0$

$\vec{E}$ (volt/metre), B (Tesla or webre m^{-2})

$\vec{j}$ (amps / m^2)

→ the first maxwell's eq. shows that either currents or time-varying electric fields may produce magnetic fields.

→ the third and fourth eq. shows that either electric charges or time varying magnetic fields may give rise to electric fields.

→ the second equation assumes that there are no magnetic monopoles and implies that a magnetic flux tube has a constant

②

strength along its length.

A fundamental supparition of normal MHD is that the electromagnetic variations are non-relativistic or 'quasi steady'.

$$V_0 << C \quad — ⑤$$

$V_0 = \frac{l_0}{t_0}$ is the characteristic / electromagnetic plasma speed.

l_0 → typical length

t_0 → typical time.

We are assuming that,

$$\frac{E_0}{l_0} \approx \frac{B_0}{t_0} \quad — ⑥$$

taking eq. ① $\nabla \times \vec{B} = \mu_0 \vec{j} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$

$$\frac{E_0}{c^2 t_0} \approx \frac{B_0 l_0}{c^2 t_0^2} = \frac{V_0^2 B_0}{c^2 l_0}$$ (Divide eq. ⑥ by $c^2 t_0$)

⇓

$$\approx \frac{V_0^2}{c^2} |\nabla \times \vec{B}|$$

⇓

this term is much smaller than $\nabla \times \vec{B}$

thus are consequence of eq. ⑤ is that the term $\frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$ may be neglected in eq. ①.

→ Another is the equation of charge continuity which is obtained from the divergence of eq. ① which is $\nabla \cdot \vec{j} = 0$

which implies physically that local accumulations in time of charge are negligible and electric currents flow in closed circuits.

A further consequence is that the ratio ③ of electrostatic to magnetic energy density,

$$\frac{E_0}{l_0} \simeq \frac{B_0}{t_0}$$

$$\frac{E_0^2}{c^2 B_0^2} \simeq \frac{l_0^2}{c^2 t_0^2}$$

$$\mu_0 \epsilon_0 \frac{E_0^2}{B_0^2} \simeq \frac{l_0^2}{c^2 t_0^2} = \frac{V_0^2}{c^2}$$

$$\boxed{\frac{\epsilon_0 E_0^2}{\frac{B_0^2}{\mu}} = \frac{V_0^2}{c^2}}$$

it is much less than the unity.

Ohm's law - Plasma moving at a non-relativistic velocity $\vec{u}$ in the presence of a magnetic field is subject to an electric field $(\vec{u} \times \vec{B})$ in addition to the electric field $\vec{E}$ which would act on material at rest. Ohm's law asserts that the current density is preportional to the total eletric field, and it may be written,

$$\vec{j} = \sigma(\vec{E} + \vec{u} \times \vec{B}) \quad -⑦$$

where σ is the electrical conductivity, measured in mho m^{-1}.

$$\sigma = \frac{ne^2 \tau}{m}$$ $\tau \rightarrow$ collision time/relaxation time.

Induction Equation →

Now taking eq. ①, ② and ⑦

$$\nabla \times \vec{B} = \mu_0 \left(\sigma(\vec{E} + \vec{u} \times \vec{B})\right) + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \nabla \times \vec{B} = \nabla \times \mu_0\left[\sigma(\vec{E} + \vec{u} \times \vec{B})\right] + \frac{1}{c^2}\nabla \times \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \nabla \times \vec{B} = \nabla \times \mu_0\left[\sigma(\vec{E} + \vec{u} \times \vec{B})\right] + \frac{1}{c^2}\frac{\partial}{\partial t}\left(-\frac{\partial \vec{B}}{\partial t}\right)$$

$$\underset{0}{\underset{\parallel}{\nabla(\nabla \cdot \vec{B})}} - \nabla^2 B = \mu_0 \sigma(\nabla \times \vec{E} + \nabla \times \vec{u} \times \vec{B}) - \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2}$$

$$-\nabla^2 B = \mu_0 \sigma(\nabla \times \vec{E} + \nabla \times \vec{u} \times \vec{B}) - \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2}$$

where $\eta = \frac{1}{(\mu\sigma)}$ is the magnetic diffusivity

Now, the Induction equation-

$$\boxed{\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \eta \nabla^2 B} \quad \text{—(8)}$$

Plasma Equations-

the plasma motion is in turn governed by equations of continuity, motion and energy. Consider first the equation of mass conservation, which may be written as;

$$\frac{d\rho}{dt} + \rho(\nabla \cdot v) = 0 \quad \text{(a)} \quad \text{—(9)}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \quad \text{(b)}$$

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla}$$

Eq. of Motion → Under the conditions of electrical neutrality, the eq. of motion may be written -

$$\rho \frac{d\vec{v}}{dt} = -\nabla p + \vec{J} \times \vec{B} + \vec{F} \quad \text{—(10)}$$

where p → plasma pressure, and the material is assumed to the subject to a plasma pressure gradient ∇p, a Lorentz force $\vec{J} \times \vec{B}$ per unit volume and a force

$$\vec{F} = \vec{F}_g + \vec{F}_v$$

effect of gravity effect of viscosity

$$\vec{F}_g = -\rho g(r)\hat{r} = -\rho \frac{G M(r)}{r^2}$$

the viscous force is,

$$\vec{F}_v = \underset{\downarrow}{\rho\nu}\left[\nabla^2\vec{v} + \frac{1}{3}\nabla(\nabla\cdot\vec{v})\right]$$

coefficient of kinematic viscosity.

It simplifies to $\vec{F}_v = \rho\nu\nabla^2 v$ if the flow is incompressible.

<u>Perfect Gas law</u> → the gas pressure is determined by an equation of state → $p = \frac{\hat{R}}{\mu}\rho T$

Fluid

- Compressible (volume changes) (density changes) ↓ By Temp. and Pressure
- Incompressible (volume does not change) (density does not change) ↓ By Temp. and Pressure

or $p = \frac{k_B}{m}\rho T$, $\rho \to \frac{1}{V}$

$pV = nk_B T$

$$\boxed{pV = RT}$$

n → total no. of particles

R → gas constant

μ → Mean atomic weight.

$\frac{D}{Dt} = \frac{d}{dt}$ ⇒ total/convective derivative for time variations following the motion.

$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho u) = 0$, it expresses the fact that the density at a point increases $\left(\frac{\partial\rho}{\partial t} > 0\right)$ if mass flows into the surrounding region $[\nabla\cdot(\rho u) < 0]$ and decreases it if it flows out.