Mathematical Expectation:

For discrete random varible X: Let $X \to x_1, x_2, \ldots x_n$. Then

$$E(X) = \sum_{i=1}^{n} x_i f(x_i)$$

For continuous random variable X:

$$E(X) = \int_{-\infty}^{\infty} x f(x)\, dx .$$

Laws of Expectation:

1. $E(aX+b) = aE(X)+b$, $a, b \to$ constants

Pf: Discrete case:

$$E(aX+b) = \sum_{i=1}^{n} (ax_i+b) f(x_i) = a\sum_{i=1}^{n} x_i f(x_i) + b\sum_{i=1}^{n} f(x_i)$$

$$= aE(X) + b \quad (\because \sum_{i=1}^{n} f(x_i) = 1.).$$

Continuous case:

$$E(aX+b) = \int_{-\infty}^{\infty} (ax+b) f(x)dx = a\int_{-\infty}^{\infty} x f(x)dx + b\int_{-\infty}^{\infty} f(x)dx$$

$$= aE(X) + b \quad (\because \int_{-\infty}^{\infty} f(x)dx = 1)$$

2. If $g(X)$ and $h(X)$ are functions of the random variable X, then $E[g(X) \pm h(X)] = E[g(X)] \pm E[h(X)]$.

Proof: Discrete case:

$$E[g(X) \pm h(X)] = \sum_{i=1}^{n} [g(x_i) \pm h(x_i)] f(x_i)$$

$$= \sum_{i=1}^{n} g(x_i) f(x_i) \pm \sum_{i=1}^{n} h(x_i) f(x_i)$$

$$= E(g(X)) \pm E(h(X))$$

Continuous case:

$$E[g(X) \pm h(X)] = \int_{-\infty}^{\infty} (g(x) \pm h(x)) f(x)\,dx$$

$$= \int_{-\infty}^{\infty} g(x) f(x)\,dx \pm \int_{-\infty}^{\infty} h(x) f(x)\,dx$$

$$= E(g(X)) \pm E(h(X))$$

Note: If $g(X) = X$ & $h(X) = Y$ then

$$E(X \pm Y) = E(X) \pm E(Y).$$

Ex Let a pair of dice be thrown and X denote the sum of numbers on them. Find expectation of X.

Soln

X	2	3	4	5	6	7	8	9	10	11	12
$f(x)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

$$\therefore E(X) = \sum_{i=1}^{n} x_i f(x_i) = \frac{2}{36} + \frac{6}{36} + \frac{12}{36} + \frac{20}{36} + \frac{30}{36} + \frac{42}{36} + \frac{40}{36} + \frac{36}{36}$$

$$+ \frac{30}{36} + \frac{22}{36} + \frac{12}{36} = \frac{252}{36} = 7.$$

Ex A random variable X has probability density function

$$f(x) = \begin{cases} 2x & 0 \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$ Find $E(X)$.

Soln $E(X) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^1 2x^2\,dx = \frac{2}{3}$