Maxwell's Equations — Lecture -01

we have electrodynamics equations in terms of divergence and curl of magnetic and electric field to represent the electromagnetic theory.

these equations are -

① $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ Gauss's law for electrostatics.

② $\nabla \cdot \vec{B} = 0$ Gauss's law for magnetism.

③ $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ Faraday's law

④ $\nabla \times \vec{B} = \mu_0 \vec{J}$ Ampere's law

we know a rule which is as the divergence of curl is always zero.

Now we are applying divergence in eq. ③

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{E}) = \vec{\nabla} \cdot \left(-\frac{\partial \vec{B}}{\partial t}\right)$$

$$0 \qquad = -\left(\frac{\partial}{\partial t}\right)(\nabla \cdot \vec{B})$$

$$= 0$$

this side is zero by the virtue of rule.

Now apply the same thing in eq. ④.

$$\vec{\nabla}\cdot(\nabla\times\vec{B}) = \vec{\nabla}\cdot(\mu_0 J)$$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \mu_0(\vec{\nabla}\cdot J)$$

Here two cases arise

① If we consider steady currents or

② If we consider non-steady currents (beyond magnetostatics problem)

for steady currents

$$\vec{\nabla}\cdot\vec{J} = 0$$

then $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = 0$

But beyond magnetostatics problem this violates the rule.

Now, Maxwell fix Ampere's law eqn.

Using continuity equation,

$$\vec{\nabla}\cdot\vec{J} = \frac{-\partial\rho}{\partial t}$$

$$= -\frac{\partial}{\partial t}(\epsilon_0 \nabla\cdot\vec{E})$$

$$\vec{\nabla}\cdot\vec{J} = -\nabla\cdot\epsilon_0\left(\frac{\partial\vec{E}}{\partial t}\right)$$

Now we add this term in Ampere's law →

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

Note → A changing magnetic field induces an electric field as well as a changing electric field induces a magnetic field.

Maxwell introduced a term which is displacement current.

$$\boxed{J_d = \epsilon_0 \frac{\partial E}{\partial t}}$$

So $\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \vec{J}_d$

this is in differential form.

Now we write this in integral form

$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc.} + \mu_0 \epsilon_0 \int \left(\frac{\partial \vec{E}}{\partial t}\right) \cdot d\vec{a}}$$

Now Maxwell's Equations with Maxwell's correction.

① $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

② $\nabla \cdot \vec{B} = 0$

③ $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

④ $\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

this is maxwell's correction term

Note → Maxwell's eqns. specify the divergence and curl of $\vec{E}$ and $\vec{B}$.