Newton's Laws of Motion and Its Applications

Newton's laws of motion are of central importance in classical physics. A large number of principles and results may be derived from Newton's laws of motion.

Definition of Force and Mass

The first two laws of motion give the concept of force and mass.
Intuitively we have some idea of mass as a measure of the "quantity of matter in an object" and loosely speaking force is a measure of the 'push' or pull on an object.
. We know that force is a vector quantity. It has magnitude as well as direction.

What causes an acceleration?

If we see that the velocity of a particle-like body changes in either direction or in magnitude, we know that something must have caused that change in velocity. From our common experience we can say that the change in velocity occurs due to the interaction of the body with the surroundings. An interaction that can cause an acceleration is known as force — the force is said to act on a body.

Units of force and mass

Standard units of mass are gram (gm) in C.G.S and Kilogram (kg) in SI unit. Standard units of force in these systems are dyne and newton.

A dyne is that force which will give a 1 gm mass an acceleration of 1 cm/s^2. A newton is that force which will give a 1 kg mass an acceleration of 1 m/s^2.

Newtonian Mechanics

The relationship between a force and the acceleration it causes was first understood by Isaac Newton. A study of that relationship as Newton presented it is called Newtonian mechanics. Newton's laws is an invaluable asset to understanding any systematic treatment of mechanics.

Validity of Newtonian Mechanics.

With the development of modern physics such that relativity and quantum mechanics, all of us know that there are some important areas of physics in which newtonian mechanics fails, while relativity and quantum mechanics succeed. Briefly newtonian mechanics breaks down for systems moving with a speed comparable to the speed of light, 3×10^8 m/sec, and it also fails for systems of atomic dimensions or smaller where quantum effects are significant. Thus it is beleived that newtonian mechanics is a special case of these two comprehensive theories (relativity and quantum mechanics). Still it is an important special case because we can apply newtonian mechanics to the motion of object ranging in size from very small (almost on the scale of atomic structure) to astronomical (object such as galaxies and cluster of galaxies).

Newton's Laws

Now we focus our attention on three primary law of motion. It is important to understand which parts of Newton's laws are based on experiment and which parts are matters of definition. In discussing the laws we must also learn how to apply them because this is essential for a real understanding of the underlying concepts.

Newton's first law of motion

Every particle persists in a state of rest or of uniform motion in a straight line (ie, with constant velocity) unless acted upon by a force.

In another words:
If the vector sum of all the forces acting on a particle is zero then and only then the particle remains unaccelerated (ie remains at rest or moves with constant velocity).

Mathematically we can write the above statement as
$\vec{a} = 0$ if and only if $\vec{F} = 0$ where $\vec{a}$ is the acceleration of the particle and $\vec{F}$ is the resultant force acting on the particle.

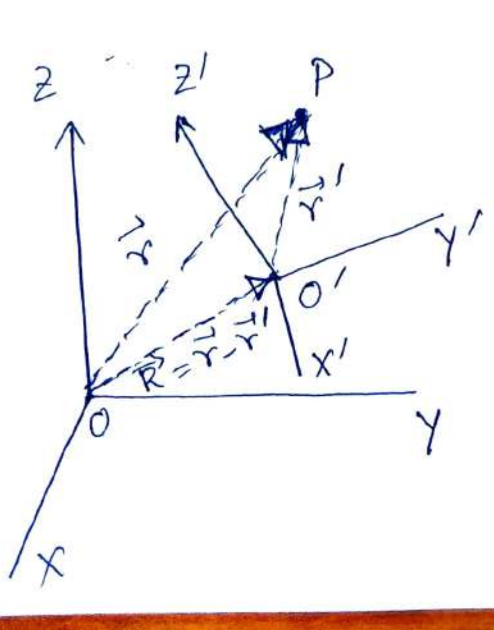
The first law of motion explains the concept of inertia, namely the application of the force that is required to move a stationary object.

Inertial frames of reference. Absolute motion:

However the concept of rest, motion or acceleration is meaningful only when a frame of reference is specified.

It must be emphasized that Newton's laws are postulated under the assumption that all measurement or observations are taken with respect to a coordinate system or frame of reference which is fixed in space i.e. is absolutely at rest. This is the so-called assumption that space or motion is absolute. It is quite clear, however, that a particle can be at rest or in uniform motion in a straight line with respect to (w.r.t) one frame of reference and be travelling in a curve and accelerating w.r.t another frame of reference.

We can show that if Newton's laws hold in one frame of reference they also hold in any other frame of reference which is moving at constant velocity relative to it. All such frames of reference are called inertial frames of reference or Newtonian frames of reference. To all observers in such inertial systems the force acting on a particle will be the same i.e, it will be invariant. This is sometimes called the classical theory of relativity.
Earth is not exactly an inertial system, but for many practical purposes can be considered as one so long as motion takes place with speed which are not too large.
Now we will show that Newton's laws of motion are same in all inertial frames of reference (i.e, the frames of reference which are moving at constant velocity relative to each other):



Two observers O and O', fixed relative to two coordinate systems OXYZ and O'X'Y'Z' respectively, observe the motion of a particle P in space. We have to show that to both the observers the particle P appears to have the same force acting on it if and only if the two coordinate systems are moving at constant velocity relative to each other.

Let the position vectors of the particle in the OXYZ and O'X'Y'Z' coordinate systems be $\vec{r}$ and $\vec{r}'$ respectively and let the position vector of O' w.r.t O be $\vec{R} = \vec{r} - \vec{r}'$.

Relative to observers O and O' the forces acting on P according to Newton's laws are given respectively by

$$\vec{F} = m\frac{d^2\vec{r}}{dt^2}, \quad \vec{F}' = m\frac{d^2\vec{r}'}{dt^2}$$

The difference in observed forces is

$$\vec{F} - \vec{F}' = m\frac{d^2}{dt^2}\left(\vec{r} - \vec{r}'\right) = m\frac{d^2\vec{R}}{dt^2} \quad \left[\text{since } \vec{R} = \vec{r} - \vec{r}'\right]$$

and this difference will be zero ie $\vec{F} - \vec{F}' = 0$ if and only if $\dfrac{d^2\vec{R}}{dt^2} = 0$ or, $\dfrac{d\vec{R}}{dt} = \text{constant}$

or, $\vec{V} = \text{constant}$ where $\vec{V}$ is velocity of O'X'Y'Z' system w.r.t. OXYZ system

ie, the coordinate systems are moving at constant velocity relative to each other. Such coordinate systems are called inertial coordinate systems.

Thus we see that if the frames are inertial (ie moving with constant velocity relative to each other) then the forces acting on the particle as observed by the two observers sitting at the origins of the two inertial systems will appear to be same ie $\vec{F} = F'$ or $\vec{F} - \vec{F}' = 0$. This implies that Newton's laws are invariant in all inertial frames of reference.

Newton's 2nd law of motion

Newton's 2nd law of motion is a mathematical formula that describes the inherent nature of force.

If $\vec{F}$ is the (external) force acting on a particle of mass m which as a consequence is moving with a velocity $\vec{v}$ then

$$\vec{F} = \frac{d}{dt}(m\vec{v}) = \frac{d\vec{p}}{dt}$$

where $\vec{p} = m\vec{v}$ is called the momentum. If m is independent of time this becomes

$$\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a} \Rightarrow \vec{a} = \frac{\vec{F}}{m}$$

where $\vec{a}$ is the acceleration of the particle ie, the force acting on the particle is the rate of change of momentum of the particle.

So a force $\vec{F}$ acting on a particle of mass m produces an acceleration $\frac{\vec{F}}{m}$ in it w.r.t. an inertial frame.

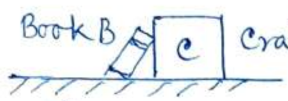
Here the force $\vec{F}$ acting on the body is the vector sum of all the forces acting on the body.

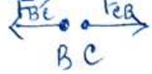
Newton's 3rd law of motion

If particle 1 acts on particle 2 with a force $\vec{F}_{12}$ in a direction along the line joining the particles, while particle 2 acts on particle 1 with a force $\vec{F}_{21}$ then $\vec{F}_{21} = -\vec{F}_{12}$.

In otherwords, to every action there is an equal and opposite reaction.

Thus the force exerted by particle 1 on particle 2 and that by particle 2 on particle 1 are equal in magnitude but opposite in direction. The forces connected by third law act on two different bodies.





For example, suppose that you position a book B so that it leans against a crate C. Then the book and the crate interact. There is a horizontal force $\vec{F}_{BC}$ on the book due to the crate C and a horizontal force $\vec{F}_{CB}$ on the crate due to the book. The pair of forces are shown in the figure.

From 3rd law of motion

$$\vec{F}_{CB} = -\vec{F}_{BC}$$

This implies that the two forces are equal in magnitude but opposite in direction.

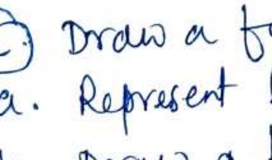
Some Applications of Newton's Laws

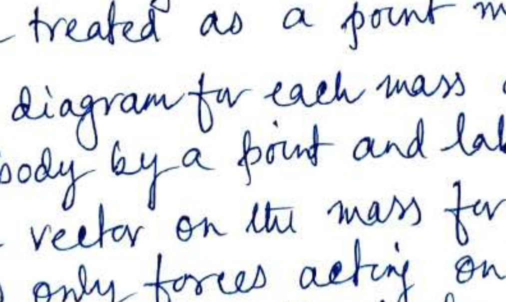
Newton's laws are meaningless equations until we know how to apply them. A number of steps are involved in analyzing a physical problem. Here are the steps:

1. Mentally divide the system into smaller systems, each of which can be treated as a point mass.

2. Draw a force diagram for each mass as follows:
   a. Represent the body by a point and label it.
   b. Draw a force vector on the mass for each force acting on it. Draw only forces acting on the body not the forces exerted by the body.

For example

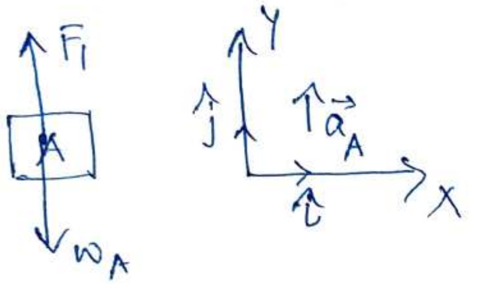




Forces on the body A   Forces on the body B.

(3.) Introduce a coordinate system :

The coordinate system must be inertial – that is, it must be fixed to an inertial frame.



Let us consider the X-Y axes which are perpendicular to each other. For instance, returning to the force diagram for block A.

Newton's second law gives

$$\vec{F}_1 + \vec{W}_A = M_A \vec{a}_A$$

Since $\vec{F}_1 = F_1 \hat{j}$, $\vec{W}_A = -W_A \hat{j}$ so we have

$$0 = M_A (a_A)_x \Rightarrow$$ the x component of the forces are zero

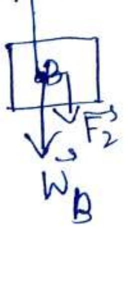
and

$$F_1 - W_A = M_A (a_A)_Y$$

So after omitting the x-component of the eq^n. of motion we simply get

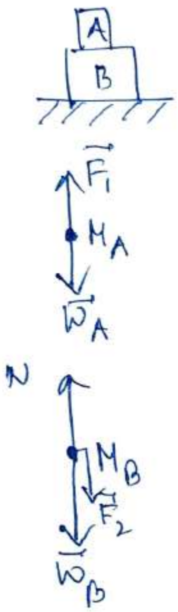
$$\boxed{F_A - W_A = M_A a_A} \longrightarrow$$ Eq^n. of motion of block A

Similarly the eq^n. of motion of body B is



$$\boxed{N - W_B - F_2 = M_B a_B} \Rightarrow$$ Eq^n. of motion of block B

(4.) If two bodies in the same system interact, the forces between them must be equal and opposite by Newton's third law. These relation should be written explicitly



For example, in the case of two blocks on the table top
$\vec{F}_1$ is the force exerted by block B on block A
$\vec{F}_2$ is the force exerted by block A on block B
Hence for Newton's 3rd law of motion

$$\vec{F}_1 = -\vec{F}_2 \quad \text{i.e., } |\vec{F}_1| = |\vec{F}_2|.$$

Note that Newton's 3rd law never relates two forces acting on the same body.

(5.) Write the constraint equations:
In many problems, bodies are constrained to move along certain paths. A pendulum bob for example moves in a circle and a block sliding on a table top is constrained to move in a plane. Each constraint can be described by a constraint equation.

As for example, the two blocks on the table top. there is no vertical acceleration and the constraint equations are

$$(\vec{a}_A)_y = 0 \quad \text{and} \quad (\vec{a}_B)_y = 0.$$

(6.) Keep track of which variables are known and which are unknown. The unknown variables can be found by solving the force equations and the constraint equations.

Completing the problem of two blocks on the table, we have

$$\left.\begin{aligned} F_1 - W_A &= M_A a_A \\ N - F_2 - W_B &= M_B a_B \end{aligned}\right\} \Rightarrow \text{Equations of motion/force equations}$$

$$\left.F_1 = F_2 \right\} \Rightarrow \text{From Newton's 3rd law}$$

$$\left.\begin{aligned} a_A &= 0 \\ a_B &= 0 \end{aligned}\right\} \Rightarrow \text{Constraint equations}$$

All that remains is the mathematical task of solving the equations. We find

$$F_1 = F_2 = W_A$$

and $N = W_A + W_B$.