

18/8/22

Date \_\_\_\_\_

Page \_\_\_\_\_

## Product or Composition in Permutation

$$\sigma = (1\ 2\ 3\ 4)(2\ 3\ 5)(1\ 2\ 4)(2\ 4\ 5)(1\ 4\ 5\ 3\ 2)$$

↓

$$(1\ 3\ 2\ 4)(5)$$

$$o(\sigma) = 4$$

### Working Rules

right to

Step 1 Start doing calculation from left.

Step 2 First of all check the image of element 1 in the right most cycle and then continuing it towards leftmost cycle.

Disjoint Cycles Two cycles are said to be disjoint if they have no common elements when they are represented in one group.

Ex → (i)  $(1\ 2\ 3\ 4)(5)$

(ii)  $(1\ 2\ 3\ 4)(5\ 6\ 7)$

$(1\ 2\ 3\ 4)(3\ 4\ 5)$  This is not disjoint cycle.

Theorem 1 "Every permutation can be written as product of disjoint cycles."

$$\text{Ex} \rightarrow (1\ 2\ 3\ 4)(3\ 4\ 5) = (1\ 2\ 3)(4\ 5)$$

Theorem 2 Let  $\sigma$  be any permutation such that  $\sigma = \sigma_1 \sigma_2 \sigma_3 \dots \sigma_k$  where  $\sigma_j, j=1, 2, 3, \dots, k$  whose  $\sigma_j$  are disjoint cycles then

$$\text{Order of } \sigma = O(\sigma) = \text{lcm}\{O(\sigma_1) O(\sigma_2) \dots O(\sigma_k)\}$$

$$\text{Ex} \rightarrow \underbrace{(1\ 2\ 3\ 4)}_{\sigma_1} \underbrace{(5\ 6\ 7)}_{\sigma_2}$$

$$O(\sigma_1) = 4$$

$$O(\sigma_2) = 3$$

$$O(\sigma) = \text{lcm}\{4, 3\} = 12 \quad \underline{\text{Ans}}$$

$$\text{ii) } \underbrace{(1\ 2\ 3)}_{\sigma_1} \underbrace{(4\ 5)}_{\sigma_2}$$

$$O(\sigma_1) = 3$$

$$O(\sigma_2) = 2$$

$$O(\sigma) = \text{lcm}\{3, 2\} = 6 \quad \underline{\text{Ans}}$$

Q. (i) Find Order of  $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 1 & 5 & 6 & 8 & 9 & 7 \end{pmatrix}$

$$\sigma = \begin{matrix} \begin{matrix} 4 \\ \uparrow \end{matrix} & \begin{matrix} 1 \\ \uparrow \end{matrix} & \begin{matrix} 1 \\ \uparrow \end{matrix} & = & \begin{matrix} 3 \\ \uparrow \end{matrix} \\ (1 & 2 & 3 & 4) & (5) & (6) & (7 & 8 & 9) \\ \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 \end{matrix}$$

$$O(\sigma) = \text{lcm}\{4, 1, 1, 3\} \\ = 12 \quad \underline{\text{Ans}}$$

(ii)  $\sigma^{24} = ?$

$$\sigma^{24} = (\sigma^{12})^2$$

$$= (I)^2$$

$$\sigma^{24} = I \quad \underline{\text{Ans}}$$

(iii)  $\sigma^{25} = \sigma^{24} \cdot \sigma$

$$= I \cdot \sigma = \sigma \quad \underline{\text{Ans}}$$

(iv)  $\sigma^{38} = (\sigma^{12})^3 \cdot \sigma^2 = I \cdot \sigma^2 = \sigma^2 \quad \underline{\text{Ans}}$

(v)  $\sigma^{2022} = (\sigma^{12})^{16} \sigma^6$

$$= I \cdot \sigma^6$$

$$= \sigma^6 \quad \underline{\text{Ans}}$$

## Inverse of a Permutation

$$\# \sigma = (a \ b \ c \ d)$$

$$\sigma^{-1} = (d \ c \ b \ a)$$

$$\# (1 \ 4 \ 2)^{-1} = (1 \ 2 \ 4)$$

$$\# (2 \ 5 \ 7)^{-1} = (2 \ 7 \ 5)$$

$$\# (1 \ 2 \ 3 \ 4)^{-1} = (4 \ 3 \ 2 \ 1)$$
$$= (1 \ 4 \ 3 \ 2)$$

### H.W. Exercise →

① Verify →  $\sigma\sigma^{-1} = I$  where  $\sigma = (a \ b \ c \ d)$

②  $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 2 & 5 & 4 \end{bmatrix}_{2 \times 5}$

Given matrix  $A$  is permutation matrix defined by  $\sigma$  then which of the following is/are correct?

Ⓐ  $A = A^{-2}$

Ⓑ  $A = A^{-1}$

Ⓒ  $A = A^{-5}$

Ⓓ  $A = A^{-11}$

Ⓔ None of these