Theorems on Conditional probability:

Law of total probability: Let $E_1, E_2, \dots E_n$ be n-mutually ~~exhaustive~~ exclusive and exhaustive events of associated with random experiment. If A is any event which occurs with E_1 or E_2 or $\dots E_n$ then

$$P(A) = \sum_{i=1}^{n} P(E_i)\, P(A/E_i) \quad , \; P(E_i) \neq 0$$

Proof: Clearly $A \subset \bigcup_{i=1}^{n} E_i$

$$\therefore A = A \cap \left(\bigcup_{i=1}^{n} E_i\right) = \bigcup_{i=1}^{n} (A \cap E_i) \quad \text{(distributive law)}$$

$\because$ $A \cap E_i \subset E_i$ & E_i's are mutually exclusive, so $\{A \cap E_i\}_{i=1}^{n}$ are also mutually exclusive.

$$\therefore P(A) = P\left(\bigcup_{i=1}^{n} A \cap E_i\right) = \sum_{i=1}^{n} P(A \cap E_i)$$

$$= \sum_{i=1}^{n} P(E_i)\, P(A/E_i) \quad // \qquad \left(\because P(A/E_i) = \frac{P(A \cap E_i)}{P(E_i)}\right)$$

Baye's Theorem: Let $E_1, E_2, \dots E_n$ ~~can~~ be n-mutually exclusive & exhaustive events of a random experiment with sample space S. Then for any event A in S

$$P(E_i/A) = \frac{P(E_i)\, P(A/E_i)}{\sum_{i=1}^{n} P(E_i) P(A/E_i)} \quad , \; P(A) \neq 0, \; P(E_i) \neq 0.$$

Proof :

$$P(E_i/A) = \frac{P(E_i \cap A)}{P(A)} \quad \text{(definition)}$$

$$= \frac{P(E_i)\,P(A/E_i)}{P(A)} \quad (\because P(A \cap B) = P(B) \cdot P(A/B))$$

$$= \frac{P(E_i)\,P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\,P(A/E_i)} \quad \text{(from law of total probability thm).}$$

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