Ex Use method of indirect proof (if necessary)

① $\neg P \wedge \neg Q \Rightarrow \neg(P \wedge Q)$

(1) $\neg P \wedge \neg Q$ Premise.
(2) $\neg(\neg(P \wedge Q))$ (assume; Method of Indirect proof)
(3) $P \wedge Q$ (2), De Morgans.
(4) P (3), Rule T.
(5) $\neg P$ (1), Rule T
(6) $P \wedge \neg P = F$ (4), (5), Rule T.
(contradiction)

Hence, our assumption was wrong $\Rightarrow$ Valid conclusion.

② $R \xrightarrow{H_1} \neg Q,\ R \vee S \xrightarrow{H_2},\ S \xrightarrow{H_3} \neg Q,\ P \xrightarrow{H_4} Q \Rightarrow \overset{C}{\neg P}$.

(1) $R \rightarrow \neg Q$ H1, Rule P
(2) $\neg R \vee \neg Q$ (1), Rule T
(3) $S \rightarrow \neg Q$ H3, Rule P
(4) $\neg S \vee \neg Q$ (3), Rule T
(5) $(\neg R \vee \neg Q) \wedge (\neg S \vee \neg Q)$ (2), (4), Rule T
(6) $(\neg R \wedge \neg S) \vee \neg Q$ (5), Rule T
(7) $\neg(R \vee S) \vee \neg Q$ (6), De Morgans law
(8) $(R \vee S) \rightarrow \neg Q$ (7), Rule T
(9) $R \vee S$ H2, Rule P
(10) $\neg Q$ (8), (9), Modus ponens
(11) $P \rightarrow Q$ H4, Rule P
(12) $\neg P$ (10), (11), Modus Tollens

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With Method of Indirect proof.

(1) P (assume)
(2) $P \rightarrow Q$ H4, Rule P
(3) Q (1), (2), Modus, ponens

(4) $S \to \neg Q$ H3, Rule P
(5) $\neg S$ (3), (4), Modus Tollens
(6) $R \to \neg Q$ H1, Rule P
(7) $\neg R$ (3), (6), Modus Tollens
(8) $\neg R \wedge \neg S$ (5), (7), Rule T
(9) $\neg(R \vee S)$ (8), De Morgans law
(10) $R \vee S$ H2, Rule P
(11) $\neg(R \vee S) \wedge (R \vee S)$
$= F$ contradiction

$\therefore$ Assumption was wrong. Hence, $\neg P$ is true (conclusion) //.

Validity of arguments:

Ex Test the validity of following arguments:

(1) If two sides of a triangle are equal, then the opposite angles are equal. Two sides of a triangle are equal. Hence, opposite angles are not equal.

Soln p: Two sides of triangle are equal
q: Opposite angles are equal.
$H_1: p \to q$ $H_2: p$ $C: \sim q$.

(1) p (H_2) Rule P
(2) $p \to q$ (H_1) Rule P
(3) q (1), (2), Rule T. (#) ∴ Invalid.

(2) I will become an engineer or a mathematician. I will not become an engineer. Therefore, I will become a mathematician

Soln: E: I will become an engineer
M: I will become a mathematician
$H_1: E \vee M$ $H_2: \sim E$ $C: M$.

(1) $E \vee M$ H_1, Rule P
(2) $\sim E$ H_2, Rule P
(3) M (1), (2), Rule $T_{//}$ ∴ Valid argument

Ex If the labour market is perfect then the wages of all persons in a particular employment will be equal. But it is always the case that the wages for such persons are not equal. Therefore the labour market is not perfect.

Soln L: Labour market is perfect W: wages of all persons in particular employment are equal

$H_1: L \to W$ $H_2: \sim W$ $C: \sim L$

(1) $L \to W$ H_1, Rule P
(2) $\sim W$ H_2, Rule P
(3) $\sim L$ (1), (2), Modus Tollens ∴ Valid argument

Ex If A wins the game then B will be happy. If C wins D will be happy. Either A or C will win. However, if A wins D will not be happy and if C wins B will not be happy. So, B will be happy iff D is not happy.

Soln $H_1: A \to B$ $H_2: C \to D$ $H_3: A \vee C$ $H_4: A \to \sim D$
$H_5: C \to \sim B$ conclusion: $B \rightleftharpoons \sim D$

(1) $C \to \sim B$ (H5), Rule P
(2) $B \to \sim C$ (1), contrapositive
(3) $A \vee C$ (H3), Rule P
(4) $\sim C \to A$ (3), Rule T
(5) $B \to A$ (2), (4), hyp. syll.
(6) $A \to \sim D$ (H4), Rule P
(7) $B \to \sim D$ (5), (6), hyp. syll
(8) $A \to B$ (H1), Rule P
(9) $\sim B \to \sim A$ (8), contrapositive
(10) $\sim A \to C$ (3), Rule T
(11) $\sim B \to C$ (9), (10), hyp. syll
(12) $C \to D$ (H2), Rule P
(13) $\sim B \to D$ (11), (12), hyp. syll
(14) $\sim D \to B$ (13), contrapositive
(15) $B \rightleftharpoons \sim D$ (7), (14), Rule T // ∴ Valid argument

Ex If A works hard then either B or C will enjoy themselves. If B enjoys himself, then A will not work hard. If D enjoys himself, then C will not. Therefore, if A works hard, D will not enjoy himself.

Soln A: A works hard
B: B enjoys himself
C: C enjoys himself
D: D enjoys himself
$H_1: A \to (B \vee C)$ $H_2: B \to \sim A$ $H_3: D \to \sim C$ $C: A \to \sim D$

Soln
(1) A — Rule CP.
(2) $A \to (B \vee C)$ — H1, Rule P
(3) $B \vee C$ — (1) (2) Cmodus ponens)
(4) $B \to \sim A$ — H2, Rule P
(5) $\sim B$ — (1) (4), Modus tollens
(6) C — (3), (5), Rule T
(7) $D \to \sim C$ — H3, Rule P
(8) $\sim D$ — (6), (7), Modus tollens
(9) $A \to \sim D$ — (1), (8), Rule T //

$\therefore$ Valid argument.

Ex If Ram misses many classes, he fails. If Ram fails then he is uneducated. If Ram reads lots of books then hes is not uneducated. Ram misses many classes and reads lots of book. Hence, he is not uneducated

Soln P: Ram misses many classes
Q: Ram fails.
R: Ram is not educated.
S: Ram reads lots of books.

$H_1: P \to Q$ $H_2: Q \to R$ $H_3: S \to \sim R$ $H_4: P \wedge S$ $C: \sim R$

Soln
(1) $P \wedge S$ — H_4, Rule P
(2) P — (1), Rule T
(3) $P \to Q$ — H_1, Rule P

(4) Q — (2), (3), Modus ponens
(5) $Q \to R$ — H_2, Rule P
(6) R — (4), (5), Modus ponens
(7) $S \to \sim R$ — H_3, Rule P
(8) $\sim S$ — (6), (7), Modus tollens
(9) S — (1), Rule T
(10) $\sim S \wedge S = F$ — (8), (9), Rule T
(Contradiction)

∴ Conclusion does not follow from given premise.
Hence, Invalid argument.

Ex: If today is sunday then there is no school. Today is either sunday or a working day. Today is not a working day. There is no party today if there is school. Therefore, there is no party today.

Soln: S: Today is Sunday Q: Today is schol R: Today is working day
P: There is party today.
$H_1: S \to \neg Q$ $H_2: S \vee R$ $H_3: \neg R$ $H_4: \neg P \rightleftharpoons Q$ $C: \neg P$

(1) $\neg R$ — H_3, Rule P
(2) $S \vee R$ — H_2, Rule P
(3) S — (1), (2), Rule T
(4) $S \to \neg Q$ — H_1, Rule P
(5) $\neg Q$ — (3), (4), Modus ponens
(6) $\neg P \rightleftharpoons Q$ — H_4, Rule P
(7) P — (5), (6), Rule T.

∴ Invalid argument.