induction the result follows :

1.6.2 Conditional Probability : Theorem of Compound Probability : La of Multiplication

The probability of an event B occurring when it is known that some event A has occu is called a *conditional probability* and it is denoted by $P(B \mid A)$. This symbol $P(B \mid$ generally read as "the probability of B, given A".

As an example, let us consider that a die is rolled. If the outcome is an odd number, t given this outcome the probability that the number is prime is the conditio probability and can be obtained as follows :

Here the sample space $S = \{1, 2, 3, 4, 5, 6\}$

Let A = the event of getting an odd number

B = the event of getting a prime number

Then,

$A = \{1, 3, 5\}, B = (2, 3, 5)$ so $A \cap B = \{3, 5\}$

, the conditional probability of event B given that A has already occurred is

$$P(B \mid A) = \frac{n(A \cap B)}{n(A)} = \frac{n(A \cap B)/n(S)}{n(A)/n(S)} = \frac{P(A \cap B)}{P(A)} = \frac{2/6}{3/6} = \frac{2}{3}$$

, $P(A \cap B)$ and $P(A)$ are found from the original sample space S. In other words, a itional probability relative to a subspace A of S may be calculated directly from the abilities assigned to the elements of the original sample space S.

tration 43 : The probability that a regularly scheduled train leaves the station on is $P(D) = 0.80$, the probability that it arrives on time is $P(A) = 0.82$; the ability that it departs and arrives on time is $P(D \cap A) = 0.75$. Find the probability a train (i) arrives on time given that it departed on time and (ii) departed on time that it has arrived on time.

tion :

The probability that a train arrives on time given that it has departed on time is

$$P(A \mid D) = \frac{P(D \cap A)}{P(D)} = \frac{0.75}{0.80}$$

The probability that a train departed on time given that it has arrived on time is

$$P(D \mid A) = \frac{P(D \cap A)}{P(A)} = \frac{0.75}{0.82}$$

tration 44 : An urn contains 6 white and 4 red balls. Two balls are drawn without acement. What is the probability that the second ball is white given that the first ball wn is white ?

ition : Let A and B be the events that the first and the second ball are white ectively. Hence,

$$P(AB) = \binom{6}{2} \Big/ \binom{10}{2} = \frac{1}{3}$$

$$P(A) = \frac{6}{10}$$

$$P(B \mid A) = \frac{P(AB)}{P(A)} = \frac{5}{9}$$

stration 45 : Let A and B events such that

$$P(A) = \frac{1}{3},\ P(B) = \frac{1}{4} \text{ and } P(A \cup B) = \frac{23}{60}$$

d $P(A \mid B)$ *and* $P(B \mid A)$.

ution : Since,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\therefore P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= \frac{1}{3} + \frac{1}{4} - \frac{23}{60}$$

$$= \frac{1}{5}$$

Thus,

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{1/5}{1/4} = \frac{4}{5}$$

$$P(B \mid A) = \frac{1/5}{1/3} = \frac{3}{5}$$

Illustration 46 : Two coins are tossed. What is the conditional ***probability*** *of getti[ng] heads given that at least one coin shows a head ?*

Solution : Here, exhaustive number of cases = 4

Now, if A denotes the event that atleast one coin shows a head and B denotes the e[vent of getting] two heads.

Then, it can be seen that A occurs in 3 ways i.e.,

$$A = \{(HT), (TH), (HH)\}$$

B occurs in 1 way *i.e.*,

$$B = \{(HH)\}$$

A and B both occur in 1 way *i.e.*,

$$A \cap B = \{(HH)\}$$

Thus,
$$P(B \mid A) = \frac{P(A \cap B)}{P(B)} = \frac{1}{3}$$

as
$$P(A) = \frac{3}{4}$$

$$P(B) = \frac{1}{4}$$

and
$$P(A \cap B) = \frac{1}{4}$$

Illustration 47 : If **A** *and* **B** *are two events such that* $P(A) = \frac{1}{3}$, $P(B) =$

$P(A \cup B) = \frac{1}{2}$

Then, find $P(A \cap B)$, $P(A \cap \overline{B})$, $P(\overline{A} \cap B)$ *and* $P(\overline{A} \cap \overline{B})$ [A

Solution : We know that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \quad P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$\Rightarrow \quad P(A \cap B) = \frac{1}{3} + \frac{1}{4} - \frac{1}{2} = \frac{4+3-6}{12} = \frac{1}{12}$$

Also,

$$P(A \cap \overline{B}) = P(A) - P(A \cap B)$$

$$= \frac{1}{3} - \frac{1}{12} = \frac{4-1}{12} = \frac{3}{12} = \frac{1}{4}$$

$$P(\overline{A} \cap B) = P(B) - P(A \cap B)$$

$$= \frac{1}{4} - \frac{1}{12} = \frac{3-1}{12} = \frac{2}{12} = \frac{1}{6}$$

And

$$P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A) - P(B) + P(A \cap B)$$

$$= 1 - \frac{1}{3} - \frac{1}{4} + \frac{1}{12}$$

$$= \frac{12-4-3+1}{12} = \frac{6}{12} = \frac{1}{2}$$

Illustration 48 : Let A and B be two events with $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ *and* $P(A \cup B) = \frac{1}{2}$.

Find

(i) $P(A \mid B)$ (ii) $P(B \mid A)$

(iii) $P(B \mid \overline{A})$ (iv) $P(A \mid \overline{B})$

(v) $P(\overline{A} \cup \overline{B})$ (vi) $P[\overline{A} \cap (A \cup B)]$

(vii) $P(\overline{A} \cup B)$ (viii) $P(\overline{A} \mid \overline{B})$

Solution : We know,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \quad P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$\Rightarrow \quad P(A \cap B) = \frac{1}{3} + \frac{1}{4} - \frac{1}{2} = \frac{1}{12}$$

(i) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{1/12}{1/4} = \frac{1}{3}$

(ii) $P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{1/12}{1/3} = \frac{1}{4}$

(iii) $P(\overline{A} \cap B) = P(B) - P(A \cap B)$

$$= \frac{1}{4} - \frac{1}{12} = \frac{1}{6}$$