

Method I

Conversion of PCNF to PDNF and vice-versa
PCNF \rightarrow PDNF

1. Let the given expression be A
2. Then A' will contain all maxterms that are not in A
3. Take $(A')'$ and use De Morgan's law on R.H.S
4. The obtained expression is in PDNF.

PDNF \rightarrow PCNF

1. Let the given expression be A
2. Then A' will contain all minterms not in A
3. Find $(A')'$ using De Morgan's law
4. Resultant is PCNF.

Ex Find PDNF of

(a) $(\sim p \vee q \vee r) \wedge (\sim p \vee q \vee \sim r) \wedge (p \vee \sim q \vee r)$

(b) $(p \vee q \vee r) \wedge (p \vee q \vee \sim r) \wedge (\sim p \vee q \vee r) \wedge (\sim p \vee q \vee \sim r)$

Soln (a) $A = (\sim p \vee q \vee r) \wedge (\sim p \vee q \vee \sim r) \wedge (p \vee \sim q \vee r)$

$$\therefore A' = (p \vee q \vee \sim r) \wedge (p \vee q \vee r) \wedge (\sim p \vee \sim q \vee r) \wedge (\sim p \vee q \vee \sim r) \wedge (\sim p \vee \sim q \vee \sim r)$$

$$(A')' = (\sim p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r)$$

or, $A = (p \wedge q \wedge r) \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge r) \vee (\sim p \wedge \sim q \wedge \sim r) \rightarrow$ PDNF

(b) $A = (p \vee q \vee r) \wedge (p \vee q \vee \sim r) \wedge (\sim p \vee q \vee r) \wedge (\sim p \vee q \vee \sim r)$

$$A' = (\sim p \vee q \vee \sim r) \wedge (p \vee \sim q \vee r) \wedge (\sim p \vee \sim q \vee r) \wedge (\sim p \vee q \vee \sim r)$$

$$\therefore (A')' = (p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r)$$

or, $A = (p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (\sim p \wedge q \wedge r) \wedge (p \wedge \sim q \wedge \sim r)$
PDNF

Ex Find PCNF of

(a) $(p \wedge q) \vee (\sim p \wedge q)$

(b) $(p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r) \vee (\sim p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge r) \vee (\sim p \wedge \sim q \wedge \sim r)$

Solⁿ (a) $A = (p \wedge q) \vee (\sim p \wedge q)$

$$A' = (p \wedge \sim q) \vee (\sim p \wedge \sim q)$$

$$\therefore (A')' = (\sim p \vee q) \wedge (p \vee q)$$

$\therefore A = (p \vee q) \wedge (\sim p \vee q) \rightarrow$ PCNF

(b) $A = (p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r) \vee (\sim p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge r) \vee (\sim p \wedge \sim q \wedge \sim r)$

$$\therefore A' = (\sim p \wedge q \wedge \sim r) \vee (p \wedge \sim q \wedge \sim r) \vee (p \wedge \sim q \wedge r)$$

$$\therefore (A')' = (p \vee \sim q \vee \sim r) \wedge (\sim p \vee \sim q \vee r) \wedge (\sim p \vee q \vee r)$$

$\therefore A = (\sim p \vee \sim r) \wedge (\sim p \vee \sim q \vee r) \wedge (p \vee \sim q \vee \sim r)$ PCNF

Ex Find PDNF of $(\sim p \rightarrow r) \wedge (p \leftrightarrow q)$ (Also see T-T done before)

Solⁿ $(\sim p \rightarrow r) \wedge (p \leftrightarrow q)$

$$\Leftrightarrow (\sim p \rightarrow r) \wedge (p \rightarrow q) \wedge (q \rightarrow p)$$

$$\Leftrightarrow (p \vee r) \wedge (\sim p \vee q) \wedge (\sim q \vee p) \quad \text{--- (1)}$$

$$\left[\begin{aligned} (p \vee r) &\Leftrightarrow (p \vee r) \vee (q \wedge \sim q) \Leftrightarrow (p \vee q \vee r) \wedge (p \vee \sim q \vee r) \\ (\sim p \vee q) &\Leftrightarrow (\sim p \vee q) \vee (r \wedge \sim r) \Leftrightarrow (\sim p \vee q \vee r) \wedge (\sim p \vee q \vee \sim r) \\ (p \vee \sim q) &\Leftrightarrow (p \vee \sim q) \vee (r \wedge \sim r) \Leftrightarrow (p \vee \sim q \vee r) \wedge (p \vee \sim q \vee \sim r) \end{aligned} \right] \quad \text{--- (2)}$$

From (1) & (2) (after removing repeated terms)

$$(\sim p \rightarrow r) \wedge (p \leftrightarrow q) \Leftrightarrow (p \vee q \vee r) \wedge (\sim p \vee q \vee r) \wedge (p \vee \sim q \vee r) \wedge (\sim p \vee q \vee \sim r) \wedge (p \vee \sim q \vee \sim r)$$

This is PCNF (which was easier to obtain). From this we will find PDNF.

$$\text{let } A = (p \vee q \vee r) \wedge (\sim p \vee q \vee r) \wedge (p \vee q \vee \sim r) \\ \wedge (\sim p \vee q \vee r) \wedge (p \vee \sim q \vee r)$$

$$\therefore A' = (p \vee \sim q \vee r) \wedge (\sim p \vee \sim q \vee r) \wedge (\sim p \vee \sim q \vee \sim r)$$

$$(A')' = (\sim p \wedge q \wedge \sim r) \vee (p \wedge q \wedge \sim r) \vee (p \wedge q \wedge r)$$

$$\therefore A = (p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (\sim p \wedge q \wedge \sim r) \rightarrow \text{PDNF}$$

Method II

For minimum $\sim p \rightarrow 0$ & $p \rightarrow 1$ $\rightarrow m_5$

$$p \wedge \sim q \wedge r \rightarrow 101 \rightarrow 5 \text{ (decimal equivalent)}$$

$$\text{similarly } \sim p \wedge q \wedge \sim r \rightarrow 010 \rightarrow 2 \rightarrow m_2$$

$$p \wedge q \wedge \sim r \rightarrow 110 \rightarrow 6 \text{ and so on}$$

For maximum $\sim p \rightarrow 1$ & $p \rightarrow 0$

$$p \vee \sim q \vee r \rightarrow 010 \rightarrow 2 \text{ (decimal eq.)} \rightarrow M_2$$

$$\sim p \vee q \vee r \rightarrow 101 \rightarrow 5 \rightarrow M_5$$

$$p \vee q \vee \sim r \rightarrow 001 \rightarrow 1 \text{ and so on}$$

$\rightarrow M_1$

\therefore For 3 variables $2^3 = 8$ minterms/maxterms are there and they take values from 0, 1, 2, ..., 7. Using this we will convert PCNF \rightarrow PDNF & vice-versa.

PCNF \Rightarrow PDNF

Convert to PDNF

$$(\sim p \vee q \vee r) \wedge (\sim p \vee \sim q \vee r) \wedge (p \vee \sim q \vee r)$$

$$= \Pi_{2,4,5}$$

$$\text{PDNF} = \Sigma_{0,1,3,6,7}$$

$$\Leftrightarrow m_0 \vee m_1 \vee m_3 \vee m_6 \vee m_7$$

$$\Leftrightarrow (\sim p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge r) \vee (\sim p \wedge q \wedge r)$$

$$\vee (p \wedge q \wedge \sim r) \vee (p \wedge q \wedge r)$$

$$\Leftrightarrow (p \wedge q \wedge r) \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge r)$$

Convert to PCNF

$$\begin{aligned}
 & (p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r) \vee (\neg p \wedge \neg q \wedge \neg r) \\
 & \quad m_7 \quad \quad m_5 \quad \quad m_2 \quad \quad m_0 \\
 & = \Sigma_{0,1,2,5,7}
 \end{aligned}$$

$$\text{PCNF} = \Pi_{3,4,6}$$

$$= M_3 \wedge M_4 \wedge M_6$$

$$\Leftrightarrow (p \vee \neg q \vee \neg r) \wedge (\neg p \vee q \vee r) \wedge (\neg p \vee \neg q \vee r)$$

$$\Leftrightarrow (\neg p \vee q \vee r) \wedge (\neg p \vee \neg q \vee r) \wedge (p \vee \neg q \vee \neg r)$$

Ex Find PCNF in three variables p, q, r of $(\neg p \wedge r) \vee (p \wedge \neg r)$

$$\Leftrightarrow [(\neg p \wedge r) \wedge (q \vee \neg q)] \vee [(p \wedge \neg r) \wedge (q \vee \neg q)]$$

$$\Leftrightarrow (\neg p \wedge q \wedge r) \vee (\neg p \wedge \neg q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r)$$

$$= \Sigma_{1,3,4,6}$$

$$\therefore \text{PCNF} = \Pi_{0,2,5,7} = M_0 \wedge M_2 \wedge M_5 \wedge M_7$$

$$= (p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg r) \wedge (\neg p \vee q \vee \neg r) \wedge (\neg p \vee \neg q \vee r)$$

Ex Find PDF in three variables p, q, r of $(p \vee q) \wedge (\neg p \vee \neg q)$

$$= (p \vee q) \wedge (\neg p \vee \neg q)$$

$$\Leftrightarrow [(p \vee q) \vee (r \wedge \neg r)] \wedge [(\neg p \vee \neg q) \vee (r \wedge \neg r)]$$

$$\Leftrightarrow (p \vee q \vee r) \wedge (p \vee q \vee \neg r) \wedge (\neg p \vee \neg q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$$

$$= \Pi_{0,1,6,7}$$

$$\therefore \text{PDF} = \Sigma_{2,3,4,5} = m_2 \vee m_3 \vee m_4 \vee m_5$$

$$\Leftrightarrow (\neg p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge q \wedge r)$$

$$\Leftrightarrow (\neg p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge q \wedge r)$$

$$\Leftrightarrow (\neg p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (p \wedge q \wedge r)$$