27/9/22

Lecture-29

Q. If $n=20$ $(U(20),\cdot)=G$ Find Factor group

$U(20)=\{1,3,7,9,11,13,17,19\}$

$U(20)=\phi(20)=\phi(2^2\cdot 5)$

$=(2^2-2)\cdot 4$

$\phi(20)=8$

Let $H=\{1,3,7,9\}$

$O(H)\,/\,O(G)$

$H<G$ & $(U(20),\cdot)$ is an abelian gp

$\Rightarrow H \Delta G$

G/H

Left Cosets of H in G

$g=1$, $1H=H$
$g=3$, $3H=H$
$g=7$, $7H=H$
$g=9$, $9H=H$
$g=11$, $11H=\{11,13,17,19\}$
$g=13$, $13H=\{13,19,11,17\}=11H$
$g=17$, $17H=\{17,11,19,13\}=11H$
$g=19$, $19H=\{19,17,13,11\}=11H$

$G/H=\{H,11H\}$ Ans

☆ $O(G/H)=2$

☆ G/H is isomorphic to Z_2

Properties of Cosets:-

H is a subgroup of G & $a*b \in G$

① $a \in aH$

let $e \in H$ is identity

$a = ae \in aH$

$\Rightarrow a \in H$ ——— Proved

or

~~②~~ $a \in a+H$

$a = e+a \in a+H$

$\Rightarrow a \in a+H$

② $aH = H$ iff $a \in H$

let $aH = H$ TPT $\rightarrow a \in H$

$\Rightarrow aH \subseteq H$ & $H \subseteq aH$

Since $ae \in aH = H$

$\Rightarrow a \in aH = H$

$\Rightarrow a \in H$

let $a \in H$ TPT $\rightarrow aH = H$

let $x \in aH$

$\Rightarrow x = ah$ $h \in H$

Now $a \in H$ and $h \in H$

$\Rightarrow ah \in H$

$x = ah \in H$

$\Rightarrow aH \subseteq H$ ——— ①

Now, let $x \in H$ and

$a \in H$, $h \in H$

$\Rightarrow ah \in H$

$\Rightarrow x \in aH$

$\Rightarrow H \subseteq aH$ —— ②

from eq-① & ②, we get →

$aH = H$

Therefore, $aH = H$ iff $a \in H$ —— Proved