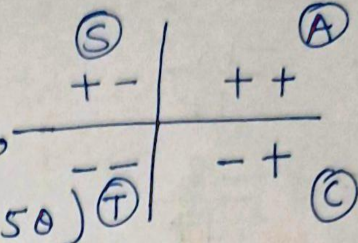
Roots of complex numbers -

Q. ① ⓐ $z^5 = -32$

In polar form

$$-32 = 32\{\cos(\pi + 2k\pi) + i\sin(\pi + 2k\pi)\}$$

let $z = r(\cos\theta + i\sin\theta)$



By De Moivre's theorem,

$$z^5 = r^5(\cos 5\theta + i\sin 5\theta)$$

$$z^5 = 32\{\cos(\pi + 2k\pi) + i\sin(\pi + 2k\pi)\}$$

$$r^5 = 32 \quad \text{so} \quad r = 2$$

$$5\theta = \pi + 2k\pi$$

$$\text{so} \quad \theta = \left(\frac{\pi + 2k\pi}{5}\right)$$

So the solution is,

$$z = 2\left\{\cos\left(\frac{\pi + 2k\pi}{5}\right) + i\sin\left(\frac{\pi + 2k\pi}{5}\right)\right\}$$

If $k = 0$ $\quad z = z_1 = 2\left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)$

$k = 1$ $\quad z = z_2 = 2\left(\cos\frac{3\pi}{5} + i\sin\frac{3\pi}{5}\right)$

$k = 2$ $\quad z = z_3 = 2\left(\cos\frac{5\pi}{5} + i\sin\frac{5\pi}{5}\right)$

$k = 3 \Rightarrow z = z_4 = 2\left(\cos\frac{7\pi}{5} + i\sin\frac{7\pi}{5}\right)$

$k = 4 \Rightarrow z = z_5 = 2\left(\cos\frac{9\pi}{5} + i\sin\frac{9\pi}{5}\right)$

Q② $\quad (-2\sqrt{3} - 2i)^{1/4} = z^{1/4}$

$$z^{1/4} = r^{1/4}\left(\cos\frac{1}{4}\theta + i\sin\frac{1}{4}\theta\right)$$

$$(-2\sqrt{3} - 2i)^{1/4} = +2^{1/4}(-\sqrt{3} - i)^{1/4}$$

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Q.②

$(-15-8i)$

$\parallel$

$17\{\cos(\theta+2k\pi)+ i\sin(\theta+2k\pi)\}$

$r=\sqrt{(15)^2+(8)^2}$

$r=\sqrt{225+64}$

$r=\sqrt{289}=17$

$\cos\theta = \frac{-15}{17}$

$\sin\theta = \frac{-8}{17}$

square root

$\sqrt{17}\cos\left(\frac{\theta}{2}+\pi\right) + \sin\left(\frac{\theta}{2}+\pi\right)$

$= -\sqrt{17}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)$

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{(1+\cos\theta)}{2}}$

$= \pm\sqrt{\frac{1-\frac{15}{17}}{2}} = \pm\frac{1}{\sqrt{17}}$

$\sin\frac{\theta}{2} = \pm\sqrt{\left(\frac{1-\cos\theta}{2}\right)}$

$= \pm\sqrt{\frac{1+\frac{15}{17}}{2}} = \pm\frac{4}{\sqrt{17}}$

$\cos\frac{\theta}{2} = \pm\frac{1}{\sqrt{17}}$

$\sin\frac{\theta}{2} = \pm\frac{4}{\sqrt{17}}$

Roots are $\rightarrow$

$(-1+4i)$ $(1-4i)$