Example 1.10 : A body having 3 m^2 of surface area is maintained at 600 K. It exchanges heat by radiation with another surface enclosing it at 300 K. Its emissivity is 0.55.

Find : (i) The rate of heat lost by radiation.

(ii) Radiation thermal resistance.

(iii) The value of equivalent convection coefficient.

Solution :

Given : $A = 3\ m^2$; $T_1 = 600$ K ; $T_2 = 300$ K ; $\in = 0.55$

(i) Rate of heat lost by radiation, Q

$$Q = \in \cdot \sigma \cdot A\left(T_1^4 - T_2^4\right)$$

$$= 0.55 \times 3 \times (5.67 \times 10^{-8})(600^4 - 300^4) = \mathbf{11366.9\ W}$$ ...Ans.

(ii) Radiation thermal resistance, R_{rad}

$$Q = \frac{(T_1 - T_2)}{R_{rad}}$$

$$\therefore\ R_{rad} = \frac{(T_1 - T_2)}{Q} = \frac{(600 - 300)}{11366.9} = \mathbf{0.0264°\ C/W}$$ ...Ans.

(iii) The value of equivalent convection coefficient, h

$$Q = h \cdot A(T_1 - T_2)$$

$$11366.9 = h \times 3 \times (600 - 300)$$

$$h = \mathbf{12.6299\ W/m^2\ K}$$ ...Ans.

Example 1.11 : Air at 27°C blows over a hot plate of 0.5 m × 1 m surface which is maintained at 227°C. The film conductance is 25 W/m^2K. There is a heat loss of 280 W by radiation from surface of the plate. The plate is 2 cm thick. Calculate :

(i) Heat transfer rate.

(ii) The temperature of the other side of the plate.

Assume thermal conductivity of plate material as 43 W/mK.

Solution :

Given : Refer Fig. P. 1.11.

Heat loss by radiation, $Q_r = 280$ W

Air (fluid) temperature, $T_\infty = 27°C$

Surface temperature, $T_w = 227°C$

Film conductance, $h = 25\ W/m^2K$,

Thickness of plate, x = 2 cm, = 0.02 m ; k = 43 W/mK,

Surface area of plate, $A = 1 \times 0.5 = 0.5\ m^2$.

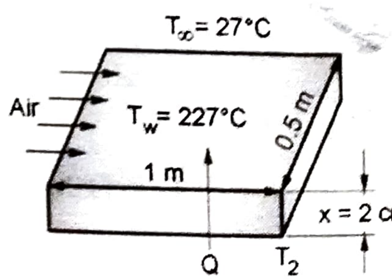

Fig. P. 1.11

(i) Heat Transfer rate :

Heat from upper surface is lost both by radiation and convection. However, other side of the plate is being heated so that the upper surface temperature of plate could be maintained at $T_w = 227°C$.

It implies that the heat conducted through the plate thickness must be equal to heat lost by convection and radiation.

Heat lost by convection,

$$Q_c = h \cdot A\,(T_w - T_\infty) = 25 \times 0.5 \times (227 - 27) = 2500 \text{ W}$$

∴ Total heat lost or heat transfer rate,

$$Q = \text{Heat lost by convection, } Q_c + \text{Heat loss by radiation, } Q_r$$

$$= 2500 + 280 = \mathbf{2780\ W}$$ ...**Ans.**

(ii) Temperature of other side of the plate, T_2

$$Q = \frac{k \cdot A \cdot (T_2 - T_w)}{x}$$

$$2780 = \frac{43 \times 0.5 \times (T_2 - 227)}{0.02} = \mathbf{229.586°C}$$ ...**Ans.**

Example 1.12 : Using approximate assumption, calculate the radiation heat transfer coefficient for a small hot surface having temperature of 152°C with emissivity of 0.85 dissipates heat by radiation into a hot surrounding area.

Solution : Surface temperature, $T_s = 152°C = 425K$, Emissivity, $\epsilon = 0.85$

Let surrounding temperature, $T_\infty = 25°C = 298$ K

The heat transfer rate by radiation from body to surroundings,

$$Q = \epsilon \cdot A \cdot \sigma \left(T_s^4 - T_\infty^4\right) \qquad ...(i)$$

Let h_r be the radiation heat transfer coefficient similar to convective heat transfer coefficient.

Then,

$$Q = h_r \cdot A\,(T_s - T_\infty) \qquad ...(ii)$$

On equating Equation (i) and (ii) we get,

$$\epsilon \cdot A \cdot \sigma \left(T_s^4 - T_o^4\right) = h_r \cdot A\,(T_s - T_\infty)$$

$$\therefore 0.85 \times A \times (5.67 \times 10^{-8})\,(425^4 - 298^4) = h_r \times A\,(425 - 298)$$

$$\therefore h_r = \mathbf{9.39\ W/m^2\,K.}$$ ...**Ans.**

Example 1.13 : A horizontal plate (k = 30 W/mK) 600 mm × 900 mm × 30mm is maintained at 300°C. The air at 30° C flows over the plate. If the convection co-efficient of air over the plate is 22 W/m^2 K and 250 W heat is lost from the plate by radiation, calculate the bottom surface temperature of the plate.

Solution : Area of plate transfering heat

$$A = 600 \text{ mm} \times 900 \text{ mm} = 54 \times 10^4 \text{ mm}^2 = \frac{54 \times 10^4}{10^6} \text{ m}^2 = 0.54 \text{ m}^2$$

Thickness of plate, $L = 30 \text{ mm} = 0.03 \text{ m}$

Surface temperature of plate, $T_s = 300°C$; Temperature of air, $T_\infty = 30°C$

$k = 30\ W/mK$; $h = 22\ W/m^2\ K$, $Q_{radiation} = 250\ W$

Let bottom surface temperature be T_1

In this case the heat is conducted through the plate which is lost from the plate surface by a combination of convection and radiation.

$$Q\ conduction = Q\ convection + Q\ radiation$$

$$\frac{k \cdot A\,(T_1 - T_s)}{L} = h \cdot A\,(T_s - T_\infty) + Q\ radiation$$

$$\therefore \frac{30 \times 0.54\,(T_1 - 30)}{0.03} = 22 \times 0.54\,(300 - 30) + 250$$

$$\therefore 540\,(T_1 - 300) = 3207.6 + 250 = 3457.6$$

$$\therefore T_1 = \mathbf{306.403°C}$$

...Ans.

Summary

- **The science of thermodynamics** deals with the energy transfers and its effect on the condition of the system while the **science of heat transfer** deals with the determination of rates of transfer of heat energy across the system.
- **Knowledge of heat transfer** helps us to determine (i) the rates of heat transfer of a given temperature difference, (ii) time taken to affect the desired amount of heat energy, (iii) surface area needed and (iv) temperature distribution across the system.
- *Heat transfer plays an important role in various industrial* **applications**. e.g. design of heat exchangers and ducts in air conditioning; cylinder and radiators in I.C. engines; combustion chambers; steam generators; condensers; transformers; furnaces; various electronic system etc.
- *Modes of heat transfer* are conduction, convection and radiation.
- **Conduction** of heat is defined as the transfer of heat from one part of substance to another part of substance by direct contact of molecules.
- The process of heat **convection** is due to the capacity of moving matter to carry heat energy.
- The heat transfer by **radiation** is due to electromagnetic radiations emitted in a wavelength band between 0.1 μ to 100 μ ($1\ \mu = 10^{-6}$ m) solely as a result of the temperature of the surface. *Such a heat transfer does not require any material medium.*
- Mechanism of heat transfer by conduction may takes place in following two ways :

  (i) **By lattice vibration** in which the energy is transferred by the collisions of molecules.

(ii) **By transport of free electrons**. They provide an energy flux in the direction of decreasing temperature, since flow of electric energy also depends on flow of free electrons, it implies that materials which are good electrical conductors are also good heat conductors.

- Amount of heat energy, Q transferred per unit time is called **heat transfer rate,** $Q\left(=\frac{Q}{\text{time}}\right)$ J/s.
- Heat transfer rate per unit area is called the **heat flux rate, q** $\left(=\frac{Q}{A}\right)$
- A system is said to have **unidirectional heat flow** if the heat transfer in other directions is negligible except along any one direction.
- A system is said to be under **steady state** if the internal energy of every element of the system remains constant and does not vary with time.
- **Fourier's law of heat conduction** states that the rate of heat flow through a simple homogeneous solid is directly proportional to the area measured normal to the direction of heat flow and the temperature gradient in the direction of heat flow i.e.

$$Q \propto A \cdot \frac{dT}{dx} \quad \text{i.e.} \quad Q = -k \cdot A \cdot \frac{dT}{dx}$$

'k' is called the coefficient of thermal conductivity having the units (W/mK).

Negative sign appears in the equation since dT is *negative in the direction of heat* flows.

- **Thermal conductivity of pure metals** decreases with increases in temperatures because the lattice vibrations impede the motion of free electrons. (**Mercury is an exception**)
- **Thermal conductivity of alloys and insulating materials,** increases with increase in temperature since they have very few free electrons and the heat transfer in them mainly depends on lattice vibrations.
- **'k' for gases** increases with increase in temperature since the number of collisions increase with increase in temperature (higher K.E.)
- **'k' for liquids** depends on pressure and temperature. It tends to decrease with increase in temperature due to decrease in density (water is an exception).
- Materials having same thermal conductivity in all the directions are called **isotropic materials** and having different 'k' in different directions are called **an-isotropic** materials.
- Heat conduction equation through a thick wall

$$\text{Heat transfer rate, } Q = \frac{k \cdot A (T_1 - T_2)}{x} \qquad \text{...(1.7)}$$

$$\text{Heat flux rate, } q = \frac{Q}{A} = \frac{k (T_1 - T_2)}{x} \qquad \text{...(1.8)}$$

- Dependence of thermal conductivity on temperature can be expressed as :
  $k = k_0 (1 + \alpha T)$

$$Q = \frac{k_0 A}{x} (T_1 - T_2) \left[1 + \frac{\alpha}{2} (T_1 + T_2) \right]$$

- Analogy between heat conduction and electrical systems.

$$Q = \frac{k A (T_1 - T_2)}{x} \frac{(T_1 - T_2)}{\left(\frac{x}{kA}\right)} = \frac{(T_1 - T_2)}{R} \; ; \; I = \frac{V}{R}$$

(i) $(T_1 - T_2)$ corresponds to potential difference, V.

(ii) Q corresponds to current flow, I.

(iii) $\left(\frac{x}{kA}\right)$ is called **thermal resistance to heat flow.** It is equivalent to electrical resistance, R.

- **Thermal diffusivity,** $\alpha = \frac{k}{\rho c}$ It indicates, how fast the heat energy propagates through a medium.
- The process of heat transfer between the solid surface and a fluid flowing past it is called **convection**. It may be **natural or free convection** or by **forced convection.**
- **The coefficient of convective heat transfer or film conductance, h** is defined as the ratio of thermal conductivity of the film to its thickness.

$$h = \frac{\text{Thermal conductivity of film, } k_f}{\text{Film thickness, } \delta}$$

value of k_f is low for gases as compared to liquids.

- **Newtons law of cooling** states that the rate of heat transfer is proportional to surface area, A and the temperature difference between the wall surface temperature, T_w and the liquid temperature T_∞ in the direction of heat flow i.e.

$$Q \propto A \cdot (T_w - T_\infty) \text{ i.e. } Q = h \cdot A (T_w - T_\infty)$$

$$Q = \frac{T_w - T_\infty}{\left(\frac{1}{h \cdot A}\right)}$$

$\left(\frac{1}{h \cdot A}\right)$ represents the thermal resistance to convective heat transfer.

- Laws of Radiation

(i) **Wien's law** states that the wavelength, λ_m corresponding to maximum energy is inversely proportional to the absolute temperature of the hot body i.e. $\lambda_m \propto \frac{1}{T}$ or $\lambda_m \cdot T = \text{constant}$.

(ii) **Kirchoff's law** states that the emissivity of the body ($\in$) at a particular temperature is equal to its absorptivity from the body at the same temperature.

(iii) **Stefan-Boltzmann law of radiation** states that the emissive power of black body is directly proportional to fourth power of its absolute temperature. i.e. $q \propto T^4$

For area A : $Q \propto A T^4$

$$Q = \sigma \cdot A \cdot T^4$$

where σ = Stefan Boltzmann's constant = 5.67×10^{-8} W/m^{2}K^4

For gray body,

$$Q = \in \cdot \sigma \cdot A \cdot T^4$$

where, $\in$ is called *emissivity of body* which is defined as the ratio of emissive power of any surface to the emissive power of a black surface at the same temperature. ($\in$ = 0 for white body, $\in$ = 1 for black body).

Exercise

[**Note** : For Answers please refer the section number indicated in bracket.]

Short answer type questions :

Q. 1 Differentiate between thermodynamics and heat transfer. **[Section 1.1]**

Q. 2 Why the knowledge of heat transfer is essential ? **[Section 1.1]**

Q. 3 State some of the applications of heat transfer. **[Section 1.1.1]**

Q. 4 What are the modes of heat transfer ? **[Section 1.2]**

Q. 5 Write a short note on "Mechanism of heat transfer". **[Section 1.3]**

Q. 6 What do you understand by unidirectional heat flow ? **[Section 1.4 (3)]**

Q. 7 State Fourier's law of heat conduction. **[Section 1.5]**

Q. 8 Define thermal conductivity of a material. **[Section 1.5.2]**

Q. 9 Why materials which are good electrical conductors are also good heat conductors ? **[Section 1.5.4 (3)]**

Q. 10 State the effect of variation of temperature on thermal conductivity in the following cases :

(a) Pure metals **[Section 1.5.4 (4)]**

(b) Alloys **[Section 1.5.4 (4)]**

(c) Gases **[Section 1.5.4 (5)]**

(d) Liquides **[Section 1.5.4 (6)]**

Q. 11 Differentiate between isotropic and anisotropic materials. **[Section 1.5.4 (7)]**

Q. 12 How does the thermal conductivity of materials vary with temperature ? **[Section 1.5.4 (8)]**

Q. 13 State the assumptions in Fourier's law of heat conduction. **[Section 1.5.4 (9)]**

Q. 14 Define coefficient of convective heat transfer **[Section 1.8.1]**

Q. 15 What do you understand by heat transfer by convection ? **[Section 1.8]**

Q. 16 State the Newton's law of cooling and define convective thermal resistance. **[Section 1.8.2]**

Q. 17 State the laws of radiation. **[Section 1.10]**

Q. 18 State the Kirchoff's and Stefan-Boltzmann law of radiations. **[Sections 1.10.2 and 1.10.3]**

Q. 19 Differentiate between black body and white body. **[Section 1.9]**

Q. 20 What is the black body ? **[Section 1.9]**

Q. 21 What is meant by one dimensional steady state heat conduction ? **[Section 1.4 (2) and (3)]**

Q. 22 Define emissivity of a body. **[Section 1.9.2]**

Q. 23 Explain the difference between free and forced convection. **[Section 1.8]**

Q. 24 Discuss the factors affecting the thermal conductivity of material. **[Section 1.5.4]**

Theory :

Q. 1 Define the terms heat, thermodynamics and heat transfer. **[Section 1.1]**

Q. 2 Name and explain the various modes of heat transfer. **[Section 1.2]**

Q. 3 Differentiate between conductive and convective heat transfers. **[Section 1.2]**

Q. 4 Write a short note on thermal conductivity of gases. **[Section 1.5.4 (5)]**

Q. 5 How does thermal conductivity of gases and liquids vary with temperature. **[Section 1.5.4 (5) and (6)]**

Q. 6 Comment upon variations of thermal conductivity of thermally conducting and insulating materials with temperature. **[Sections 1.5.4 (3) and 1.5.4 (4)]**

Q. 7 Explain in brief the analogy between the heat flow and electricity with its significance. **[Section 1.7]**

. 8 Discuss the mechanism of thermal conduction in gases and solids. Name some good conductors and poor conductors of heat. **[Section 1.3 and 1.5.4 (3)]**

. 9 Write short notes on :

(i) Concept of black body. **[Section 1.9]**

(ii) Newton's law of cooling by convection. **[Section 1.9.2]**

. 10 Write short note on "Stefan-Boltzmann law" **[Section 1.10.3]**

. 11 What are boundary condition for conduction ? **[Section 1.6]**

. 12 Discuss Fourier's law of heat conduction, why negative sign inserted in its expression. **[Section 1.6]**

. 13 State Kirchoff's law. **[Section 1.10.2]**

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