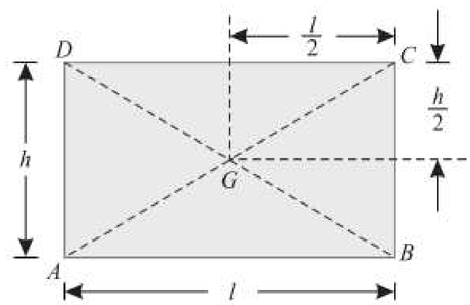
4. CENTROID AND MOMENT OF INERTIA

CENTRE OF GRAVITY: The point, through which the whole weight of the body acts, irrespective of its position, is known as centre of gravity (briefly written as C.G.). It may be noted that everybody has one and only one centre of gravity.

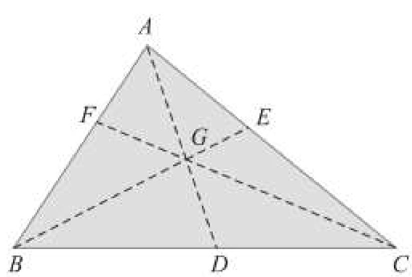
CENTROID: The plane figures (like triangle, quadrilateral, circle etc.) have only areas, but no mass. The centre of area of such figures is known as centroid. The method of finding out the centroid of a figure is the same as that of finding out the centre of gravity of a body.

CENTRE OF GRAVITY BY GEOMETRICAL CONSIDERATIONS:
The centre of gravity of simple figures may be found out from the geometry of the figure as given below.

1. The centre of gravity of uniform rod is at its middle point.
2. The centre of gravity of a rectangle (or a parallelogram) is at the point, where its diagonals meet each other. It is also a middle point of the length as well as the breadth of the rectangle as shown in Fig.



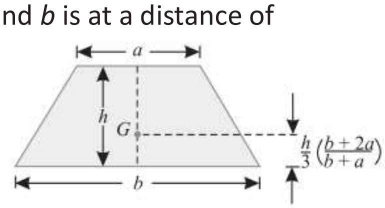
3. The centre of gravity of a triangle is at the point, where the three medians (a median is a line connecting the vertex and middle point of the opposite side) of the triangle meet as shown in Fig.



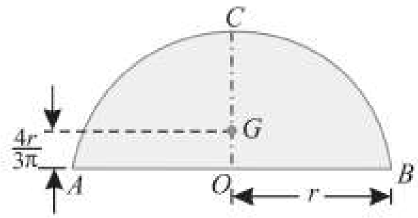
4. The centre of gravity of a trapezium with parallel sides *a* and *b* is at a distance of

$$\frac{h}{3}\times\left(\frac{b+2a}{b+a}\right)$$

measured form the side *b* as shown in Fig.



5. The centre of gravity of a semicircle is at a distance of 4r/3 π from its base measured along the vertical radius as shown in Fig.



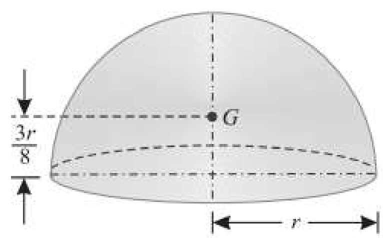
6. The centre of gravity of a circular sector making semi-vertical angle α is at a distance of

$$\frac{2r}{3}\frac{\sin\alpha}{\alpha}$$

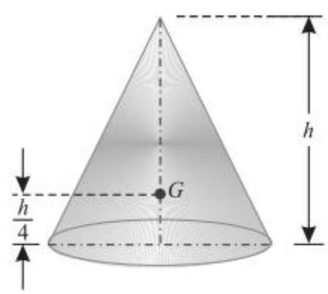
7. The centre of gravity of a cube is at a distance of *l*/ 2 from every face (where *l* is the length of each side).

8. The centre of gravity of a sphere is at a distance of *d* /2 from every point (where *d* is the diameter of the sphere).

9. The centre of gravity of a hemisphere is at a distance of 3*r*/ 8 from its base, measured along the vertical radius as shown in Fig.



10. The centre of gravity of right circular solid cone is at a distance of *h* /4 from its base, measured along the vertical axis as shown in Fig.



AXIS OF REFERENCE:

The centre of gravity of a body is always calculated with reference to some assumed axis known as axis of reference (or sometimes with reference to some point of reference). The axis of reference, of plane figures, is generally taken as the lowest line of the figure for calculating $\bar{y}$ and the left line of the figure for calculating $\bar{x}$.

CENTRE OF GRAVITY OF PLANE FIGURES:

Let $\bar{x}$ and $\bar{y}$ be the co-ordinates of the centre of gravity with respect to some axis of reference, then

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3 + \ldots\ldots}{a_1 + a_2 + a_3}$$

and

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3 + \ldots\ldots}{a_1 + a_2 + a_3 + \ldots}$$

where a_1, a_2, a_3........ etc., are the areas into which the whole figure is divided x_1, x_2, x_3 ..... etc., are the respective co-ordinates of the areas a_1, a_2, a_3....... on X-X axis with respect to same axis of reference.

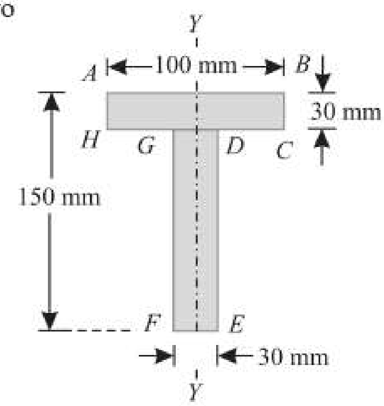
y_1, y_2, y_3....... etc., are the respective co-ordinates of the areas a_1, a_2, a_3....... on Y-Y axis with respect to same axis of the reference.

CENTRE OF GRAVITY OF SYMMETRICAL SECTIONS:

Sometimes, the given section, whose centre of gravity is required to be found out, is symmetrical about X-X axis or Y-Y axis. In such cases, the procedure for calculating the centre of gravity of the body is very much simplified; as we have only to calculate either $\bar{x}$ or $\bar{y}$. This is due to the reason that the centre of gravity of the body will lie on the axis of symmetry.

EXAMPLE: Find the centre of gravity of a 100 mm × 150 mm × 30 mm T-section.

Solution. As the section is symmetrical about Y-Y axis, bisecting the web, therefore its centre of gravity will lie on this axis. Split up the section into two rectangles *ABCH* and *DEFG* as shown in Fig 6.10.



Let bottom of the web *FE* be the axis of reference.

(*i*) *Rectangle ABCH*

$$a_1 = 100 \times 30 = 3000 \text{ mm}^2$$

and

$$y_1 = \left(150 - \frac{30}{2}\right) = 135 \text{ mm}$$

(*ii*) *Rectangle DEFG*

$$a_2 = 120 \times 30 = 3600 \text{ mm}^2$$

and

$$y_2 = \frac{120}{2} = 60 \text{ mm}$$

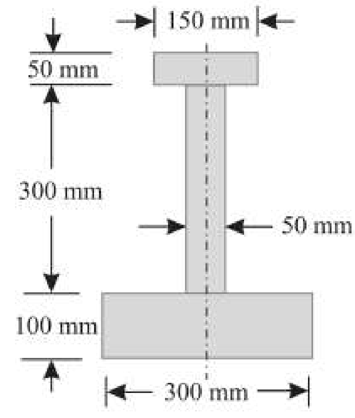
We know that distance between centre of gravity of the section and bottom of the flange *FE*,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(3000 \times 135) + (3600 \times 60)}{3000 + 3600} \text{ mm}$$

$$= 94.1 \text{ mm} \qquad \textbf{Ans.}$$

EXAMPLE: An I-section has the following dimensions in mm units:
Bottom flange = 300 × 100
Top flange = 150 × 50
Web = 300 × 50
Determine mathematically the position of centre of gravity of the section.

Solution. As the section is symmetrical about *Y-Y* axis, bisecting the web, therefore its centre of gravity will lie on this axis. Now split up the section into three rectangles as shown in Fig.



Let bottom of the bottom flange be the axis of reference.

(*i*) *Bottom flange*

$$a_1 = 300 \times 100 = 30\,000 \text{ mm}^2$$

and $$y_1 = \frac{100}{2} = 50 \text{ mm}$$

(*ii*) *Web*

$$a_2 = 300 \times 50 = 15\,000 \text{ mm}^2$$

and $$y_2 = 100 + \frac{300}{2} = 250 \text{ mm}$$

(*iii*) *Top flange*

$$a_3 = 150 \times 50 = 7500 \text{ mm}^2$$

and $$y_3 = 100 + 300 + \frac{50}{2} = 425 \text{ mm}$$

We know that distance between centre of gravity of the section and bottom of the flange,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(30\,000 \times 50) + (15\,000 \times 250) + (7500 \times 425)}{30\,000 + 15\,000 + 7500} = 160.7 \text{ mm} \quad \textbf{Ans.}$$

CENTRE OF GRAVITY OF UNSYMMETRICAL SECTIONS:

Sometimes, the given section, whose centre of gravity is required to be found out, is not symmetrical either about *X-X* axis or *Y-Y* axis. In such cases, we have to find out both the values of $\bar{x}$ and $\bar{y}$

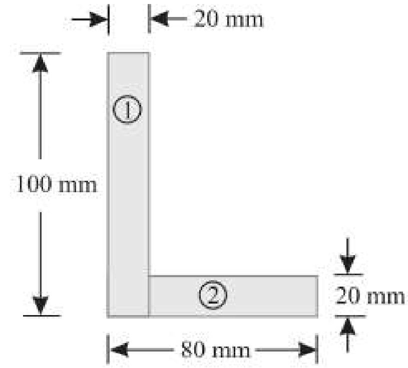
EXAMPLE: Find the centroid of an unequal angle section 100 mm × 80 mm × 20 mm.

Solution. As the section is not symmetrical about any axis, therefore we have to find out the values of $\bar{x}$ and $\bar{y}$ for the angle section. Split up the section into two rectangles as shown in Fig.

Let left face of the vertical section and bottom face of the horizontal section be axes of reference.

Let left face of the vertical section and bottom face of the horizontal section be axes of reference.

(*i*) *Rectangle* 1



$$a_1 = 100 \times 20 = 2000 \text{ mm}^2$$

$$x_1 = \frac{20}{2} = 10 \text{ mm}$$

and $$y_1 = \frac{100}{2} = 50 \text{ mm}$$

(*ii*) *Rectangle* 2

$$a_2 = (80 - 20) \times 20 = 1200 \text{ mm}^2$$

$$x_2 = 20 + \frac{60}{2} = 50 \text{ mm}$$

and $$y_2 = \frac{20}{2} = 10 \text{ mm}$$

We know that distance between centre of gravity of the section and left face,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(2000 \times 10) + (1200 \times 50)}{2000 + 1200} = 25 \text{ mm} \quad \textbf{Ans.}$$

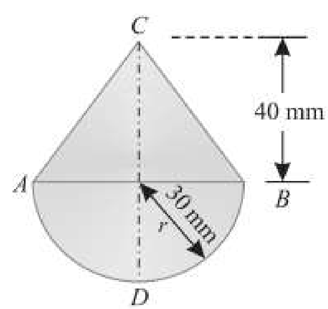
Similarly, distance between centre of gravity of the section and bottom face,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{} = \frac{(2000 \times 50) + (1200 \times 10)}{} = 35 \text{ mm} \quad \textbf{Ans.}$$

EXAMPLE: A body consists of a right circular solid cone of height 40 mm and radius 30 mm placed on a solid hemisphere of radius 30 mm of the same material. Find the position of centre of gravity of the body.

Solution. As the body is symmetrical about Y-Y axis, therefore its centre of gravity will lie on this axis as shown in Fig. Let bottom of the hemisphere (D) be the point of reference.

(*i*) *Hemisphere*



$$v_1 = \frac{2\pi}{3} \times r^3 = \frac{2\pi}{3}(30)^3 \text{ mm}^3$$

$$= 18\,000\,\pi \text{ mm}^3$$

and $$y_1 = \frac{5r}{8} = \frac{5 \times 30}{8} = 18.75 \text{ mm}$$

(*ii*) *Right circular cone*

$$v_2 = \frac{\pi}{3} \times r^2 \times h = \frac{\pi}{3} \times (30)^2 \times 40 \text{ mm}^3$$

$$= 12\,000\,\pi \text{ mm}^3$$

and $$y_2 = 30 + \frac{40}{4} = 40 \text{ mm}$$

We know that distance between centre of gravity of the body and bottom of hemisphere D,

$$\bar{y} = \frac{v_1 y_1 + v_2 y_2}{v_1 + v_2} = \frac{(18\,000\pi \times 18.75) + (12\,000\pi \times 40)}{18\,000\pi + 12\,000\pi} \text{ mm}$$

$$= 27.3 \text{ mm} \quad \textbf{Ans.}$$

MOMENT OF INERTIA: The moment of a force (P) about a point, is the product of the force and perpendicular distance (x) between the point and the line of action of the force (*i.e*. $P.x$). This moment is also called first moment of force. If this moment is again multiplied by the perpendicular distance (x) between the point and the line of action of the force *i.e*. $P.x$ (x) = $Px2$, then this quantity is called moment of the moment of a force or second moment of force or moment of inertia (briefly written as M.I.).

MOMENT OF INERTIA OF A PLANE AREA:

Consider a plane area, whose moment of inertia is required to be found out. Split up the whole area into a number of small elements.

Let $a_1, a_2, a_3, \ldots =$ Areas of small elements, and

$r_1, r_2, r_3, \ldots =$ Corresponding distances of the elements from the line about which the moment of inertia is required to be found out.

Now the moment of inertia of the area,

$$I = a_1 r_1^2 + a_2 r_2^2 + a_3 r_3^2 + \ldots$$
$$= \Sigma\, a\, r^2$$

UNITS OF MOMENT OF INERTIA:

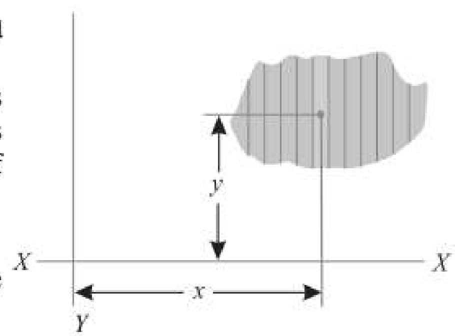
As a matter of fact the units of moment of inertia of a plane area depend upon the units of the area and the length. e.g.

1. If area is in m^2 and the length is also in m, the moment of inertia is expressed in m^4
2. If area in mm^2 and the length is also in mm, then moment of inertia is expressed in mm^4.

MOMENT OF INERTIA BY INTEGRATION:

The moment of inertia of an area may also be found out by the method of integration as discussed below:

Consider a plane figure, whose moment of inertia is required to be found out about X-X axis and Y-Y axis as shown in Fig Let us divide the whole area into a no. of strips. Consider one of these strips.



. Moment of inertia by integration.

Let $dA =$ Area of the strip

$x =$ Distance of the centre of gravity of the strip on X-X axis and

$y =$ Distance of the centre of gravity of the strip on Y-Y axis.

We know that the moment of inertia of the strip about Y-Y axis

$$= dA \,.\, x^2$$

Now the moment of inertia of the whole area may be found out by integrating above equation. *i.e.*,

$$I_{YY} = \Sigma\, dA \,.\, x^2$$

Similarly $\quad I_{XX} = \Sigma\, dA \,.\, y^2$

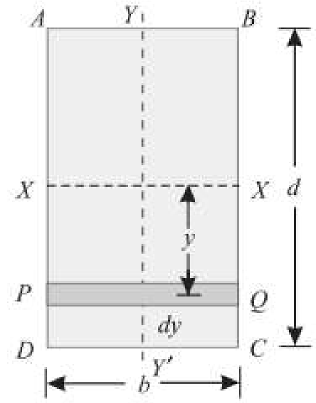
MOMENT OF INERTIA OF A RECTANGULAR SECTION:

Consider a rectangular section *ABCD* as shown in Fig. whose moment of inertia is required to be found out.

Let b = Width of the section and

d = Depth of the section.

Now consider a strip *PQ* of thickness dy parallel to *X-X* axis and at a distance y from it as shown in the figure



Rectangular section.

$\therefore$ Area of the strip

$= b.dy$

We know that moment of inertia of the strip about *X-X* axis,

$= \text{Area} \times y^2 = (b.\ dy)\ y^2 = b.\ y^2.\ dy$

Now *moment of inertia of the whole section may be found out by integrating the above equation for the whole length of the lamina *i.e.* from $-\frac{d}{2}$ to $+\frac{d}{2}$,

$$I_{xx} = \int_{-\frac{d}{2}}^{+\frac{d}{2}} b.y^2.dy = b\int_{-\frac{d}{2}}^{+\frac{d}{2}} y^2\ .\ dy$$

$$= b\left[\frac{y^3}{3}\right]_{-\frac{d}{2}}^{+\frac{d}{2}} = b\left[\frac{(d/2)^3}{3} - \frac{(-d/2)^3}{3}\right] = \frac{bd^3}{12}$$

Similarly, $I_{YY} = \dfrac{db^3}{12}$

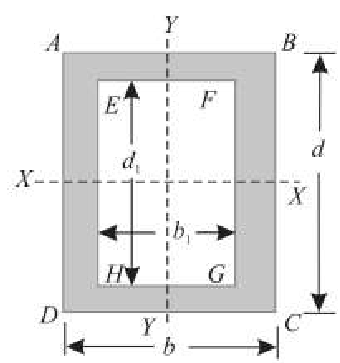
MOMENT OF INERTIA OF A HOLLOW RECTANGULAR SECTION:

Consider a hollow rectangular section, in which *ABCD* is the main section and *EFGH* is the cut out section as shown in Fig

Let b = Breadth of the outer rectangle,

d = Depth of the outer rectangle and

b_1, d_1 = Corresponding values for the cut out rectangle.



Hollow rectangular section.

We know that the moment of inertia, of the outer rectangle *ABCD* about *X-X* axis

$$= \frac{bd^3}{12} \qquad ...(i)$$

and moment of inertia of the cut out rectangle *EFGH* about *X-X* axis

$$= \frac{b_1 d_1^3}{12} \qquad ...(ii)$$

$\therefore$ M.I. of the hollow rectangular section about *X-X* axis,

I_{XX} = M.I. of rectangle *ABCD* – M.I. of rectangle *EFGH*

$$= \frac{bd^3}{12} - \frac{b_1 d_1^3}{12}$$

Similarly, $I_{yy} = \dfrac{db^3}{12} - \dfrac{d_1 b_1^3}{12}$

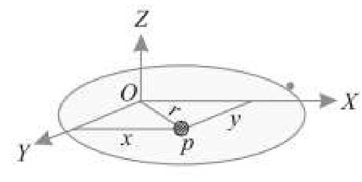
THEOREM OF PERPENDICULAR AXIS:

It states, *If I_{XX} and I_{YY} be the moments of inertia of a plane section about two perpendicular axis meeting at O, the moment of inertia I_{ZZ} about the axis Z-Z, perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by:*

$$I_{ZZ} = I_{XX} + I_{YY}$$

Proof :

Consider a small lamina (P) of area da having co-ordinates as x and y along OX and OY two mutually perpendicular axes on a plane section as shown in Fig.



Theorem of perpendicular axis.

Now consider a plane OZ perpendicular to OX and OY. Let (r) be the distance of the lamina (P) from Z-Z axis such that $OP = r$.

From the geometry of the figure, we find that

$$r^2 = x^2 + y^2$$

We know that the moment of inertia of the lamina P about X-X axis,

$$I_{XX} = da.\, y^2 \qquad \text{...}[\because I = \text{Area} \times (\text{Distance})^2]$$

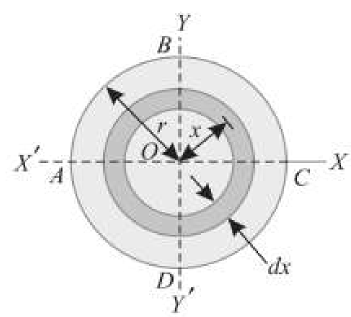
Similarly, $$I_{YY} = da.\, x^2$$

and $$I_{ZZ} = da.\, r^2 = da\,(x^2 + y^2) \qquad \text{...}(\because r^2 = x^2 + y^2)$$

$$= da.\, x^2 + da.\, y^2 = I_{YY} + I_{XX}$$

MOMENT OF INERTIA OF A CIRCULAR SECTION:

Consider a circle $ABCD$ of radius (r) with centre O and X-X' and Y-Y' be two axes of reference through O as shown in Fig.



Circular section.

Now consider an elementary ring of radius x and thickness dx. Therefore area of the ring,

$$da = 2\,\pi\, x.\, dx$$

and moment of inertia of ring, about X-X axis or Y-Y axis

$$= \text{Area} \times (\text{Distance})^2$$
$$= 2\,\pi\, x.\, dx \times x^2$$
$$= 2\,\pi\, x^3.\, dx$$

Now moment of inertia of the whole section, about the central axis, can be found out by integrating the above equation for the whole radius of the circle *i.e.*, from 0 to r.

$$\therefore \qquad I_{ZZ} = \int_0^r 2\pi x^3\,.\,dx = 2\pi\int_0^r x^3\,.\,dx$$

$$I_{ZZ} = 2\pi\left[\frac{x^4}{4}\right]_0^r = \frac{\pi}{2}(r)^4 = \frac{\pi}{32}(d)^4 \qquad \text{...}\left(\text{substituting } r = \frac{d}{2}\right)$$

We know from the Theorem of Perpendicular Axis that

$$I_{XX} + I_{YY} = I_{ZZ}$$

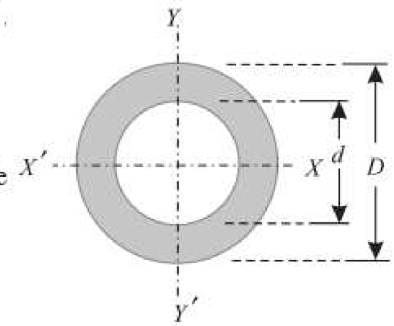
$$* I_{XX} = I_{YY} = \frac{I_{ZZ}}{2} = \frac{1}{2} \times \frac{\pi}{32}(d)^4 = \frac{\pi}{64}(d)^4$$

MOMENT OF INERTIA OF A HOLLOW CIRCULAR SECTION:

Consider a hollow circular section as shown in Fig. whose moment of inertia is required to be found out.

Let D = Diameter of the main circle, and

d = Diameter of the cut out circle.



Hollow circular section.

We know that the moment of inertia of the main circle about X-X axis

$$= \frac{\pi}{64}(D)^4$$

and moment of inertia of the cut-out circle about X-X axis

$$= \frac{\pi}{64}(d)^4$$

∴ Moment of inertia of the hollow circular section about X-X axis,

I_{XX} = Moment of inertia of main circle – Moment of inertia of cut out circle,

$$= \frac{\pi}{64}(D)^4 - \frac{\pi}{64}(d)^4 = \frac{\pi}{64}(D^4 - d^4)$$

Similarly, $$I_{YY} = \frac{\pi}{64}(D^4 - d^4)$$

THEOREM OF PARALLEL AXIS:

It states, *If the moment of inertia of a plane area about an axis through its centre of gravity is denoted by I_G, then moment of inertia of the area about any other axis* ***AB****, parallel to the first, and at a distance h from the centre of gravity is given by:*

$$I_{AB} = I_G + ah^2$$

where I_{AB} = Moment of inertia of the area about an axis AB,

I_G = Moment of Inertia of the area about its centre of gravity

a = Area of the section, and

h = Distance between centre of gravity of the section and axis AB.

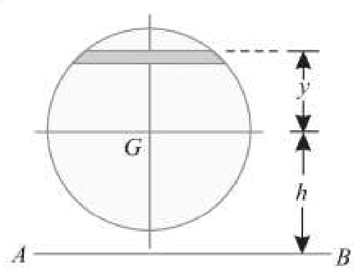
Proof

Consider a strip of a circle, whose moment of inertia is required to be found out about a line AB as shown in Fig.

Let δa = Area of the strip

y = Distance of the strip from the centre of gravity the section and

h = Distance between centre of gravity of the section and the axis AB.



Theorem of parallel axis.

We know that moment of inertia of the whole section about an axis passing through the centre of gravity of the section

$$= \delta a.\, y^2$$

and moment of inertia of the whole section about an axis passing through its centre of gravity,

$$I_G = \Sigma\, \delta a .\, y^2$$

$\therefore$ Moment of inertia of the section about the axis AB,

$$I_{AB} = \Sigma\, \delta a\, (h + y)^2 = \Sigma\, \delta a\, (h^2 + y^2 + 2\, h\, y)$$

$$= (\Sigma\, h^2 .\, \delta a) + (\Sigma\, y^2 .\, \delta a) + (\Sigma\, 2\, h\, y\, .\, \delta a)$$

$$= a\, h^2 + I_G + 0$$

It may be noted that $\Sigma\, h^2 .\, \delta a = a\, h^2$ and $\Sigma\, y^2 .\, \delta a = I_G$ [as per equation (*i*) above] and $\Sigma\, \delta a . y$ is the algebraic sum of moments of all the areas, about an axis through centre of gravity of the section and is equal to $a . \bar{y}$, where $\bar{y}$ is the distance between the section and the axis passing through the centre of gravity, which obviously is zero.

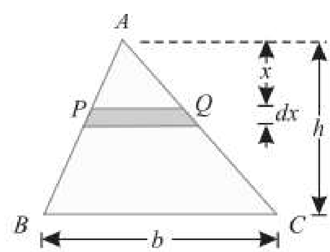
MOMENT OF INERTIA OF A TRIANGULAR SECTION:

Consider a triangular section ABC whose moment of inertia is required to be found out.

Let b = Base of the triangular section and

h = Height of the triangular section.

Now consider a small strip PQ of thickness dx at a distance of x from the vertex A as shown in Fig. From the geometry of the figure, we find that the two triangles APQ and ABC are similar. Therefore



. Triangular section.

$$\frac{PQ}{BC} = \frac{x}{h} \quad \text{or} \quad PQ = \frac{BC.x}{h} = \frac{bx}{h} \qquad (\because BC = \text{base} = b)$$

We know that area of the strip PQ

$$= \frac{bx}{h} .\, dx$$

and moment of inertia of the strip about the base BC

$$= \text{Area} \times (\text{Distance})^2 = \frac{bx}{h}\, dx\, (h - x)^2 = \frac{bx}{h}\, (h - x)^2\, dx$$

Now moment of inertia of the whole triangular section may be found out by integrating the above equation for the whole height of the triangle *i.e.*, from 0 to h.

$$I_{BC} = \int_0^h \frac{b\, x}{h}\, (h - x)^2\, dx$$

$$= \frac{b}{h} \int_0^h x\,(h^2 + x^2 - 2hx)\,dx$$

$$= \frac{b}{h} \int_0^h (xh^2 + x^3 - 2hx^2)\,dx$$

$$= \frac{b}{h} \left[\frac{x^2 h^2}{2} + \frac{x^4}{4} - \frac{2hx^3}{3} \right]_0^h = \frac{bh^3}{12}$$

We know that distance between centre of gravity of the triangular section and base BC,

$$d = \frac{h}{3}$$

$\therefore$ Moment of inertia of the triangular section about an axis through its centre of gravity and parallel to X-X axis,

$$I_G = I_{BC} - ad^2 \qquad \ldots(\because\ I_{XX} = I_G + a\,h^2)$$

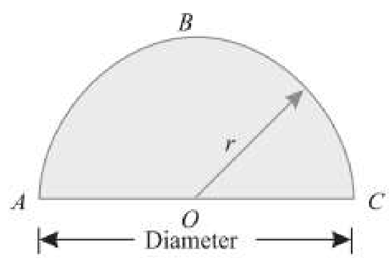
$$= \frac{bh^3}{12} - \left(\frac{bh}{2}\right)\left(\frac{h}{3}\right)^2 = \frac{bh^3}{36}$$

MOMENT OF INERTIA OF A SEMICIRCULAR SECTION:

Consider a semicircular section ABC whose moment of inertia is required to be found out as shown in Fig.

Let r = Radius of the semicircle.

We know that moment of inertia of the semicircular section about the base AC is equal to half the moment of inertia of the circular section about AC. Therefore moment of inertia of the semicircular section ABC about the base AC,



Semicircular section ABC.

$$I_{AC} = \frac{1}{2} \times \frac{\pi}{64} \times (d)^4 = 0.393\, r^4$$

We also know that area of semicircular section,

$$a = \frac{1}{2} \times \pi r^2 \ \frac{\pi r^2}{2}$$

and distance between centre of gravity of the section and the base AC,

$$h = \frac{4r}{3\pi}$$

$\therefore$ Moment of inertia of the section through its centre of gravity and parallel to x-x axis,

$$I_G = I_{AC} - ah^2 = \left[\frac{\pi}{8} \times (r)^4\right] - \left[\frac{\pi r^2}{2}\left(\frac{4r}{3\pi}\right)^2\right]$$

$$= \left[\frac{\pi}{8} \times (r)^4\right] - \left[\frac{8}{9\pi} \times (r)^4\right] = 0.11\, r^4$$

Note. The moment of inertia about y-y axis will be the same as that about the base AC *i.e.*, $0.393\, r^4$.

MOMENT OF INERTIA OF A COMPOSITE SECTION:

The moment of inertia of a composite section may be found out by the following steps :

1. First of all, split up the given section into plane areas (*i.e.*, rectangular, triangular, circular etc., and find the centre of gravity of the section).
2. Find the moments of inertia of these areas about their respective centres of gravity.
3. Now transfer these moment of inertia about the required axis (AB) by the Theorem of Parallel Axis, *i.e.*,

$$I_{AB} = I_G + ah^2$$

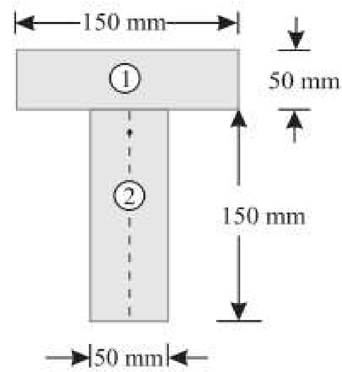
where I_G = Moment of inertia of a section about its centre of gravity and parallel to the axis.
a = Area of the section,
h = Distance between the required axis and centre of gravity of the section.

4. The moments of inertia of the given section may now be obtained by the algebraic sum of the moment of inertia about the required axis.

EXAMPLE: Find the moment of inertia of a T-section with flange as 150 mm × 50 mm and web as 150 mm × 50 mm about X-X and Y-Y axes through the centre of gravity of the section.

Solution. The given T-section is shown in Fig.

First of all, let us find out centre of gravity of the section. As the section is symmetrical about Y-Y axis, therefore its centre of gravity will lie on this axis. Split up the whole section into two rectangles *viz*., 1 and 2 as shown in figure. Let bottom of the web be the axis of reference.



(*i*) *Rectangle* (1)

$$a_1 = 150 \times 50 = 7500 \text{ mm}^2$$

and $$y_1 = 150 + \frac{50}{2} = 175 \text{ mm}$$

(*ii*) *Rectangle* (2)

$$a_2 = 150 \times 50 = 7500 \text{ mm}^2$$

and $$y_2 = \frac{150}{2} = 75 \text{ mm}$$

We know that distance between centre of gravity of the section and bottom of the web,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(7500 \times 175) + (7500 \times 75)}{7500 + 7500} = 125 \text{ mm}$$

Moment of inertia about X-X axis

We also know that M.I. of rectangle (1) about an axis through its centre of gravity and parallel to X-X axis.

$$I_{G1} = \frac{150\,(50)^3}{12} = 1.5625 \times 10^6 \text{ mm}^4$$

and distance between centre of gravity of rectangle (1) and X-X axis,

$$h_1 = 175 - 125 = 50 \text{ mm}$$

∴ Moment of inertia of rectangle (1) about *X-X* axis

$$I_{G1} + a_1 h_1^2 = (1.5625 \times 10^6) + [7500 \times (50)^2] = 20.3125 \times 10^6 \text{ mm}^4$$

Similarly, moment of inertia of rectangle (2) about an axis through its centre of gravity and parallel to *X-X* axis,

$$I_{G2} = \frac{50\,(150)^3}{12} = 14.0625 \times 10^6 \text{ mm}^4$$

and distance between centre of gravity of rectangle (2) and *X-X* axis,

$$h_2 = 125 - 75 = 50 \text{ mm}$$

∴ Moment of inertia of rectangle (2) about *X-X* axis

$$= I_{G2} + a_2 h_2^2 = (14.0625 \times 10^6) + [7500 \times (50)^2] = 32.8125 \times 10^6 \text{ mm}^4$$

Now moment of inertia of the whole section about *X-X* axis,

$$I_{XX} = (20.3125 \times 10^6) + (32.8125 \times 10^6) = 53.125 \times 10^6 \text{ mm}^4 \quad \textbf{Ans.}$$

Moment of inertia about Y-Y axis

We know that M.I. of rectangle (1) about *Y-Y* axis

$$= \frac{50\,(150)^3}{12} = 14.0625 \times 10^6 \text{ mm}^4$$

and moment of inertia of rectangle (2) about *Y-Y* axis,

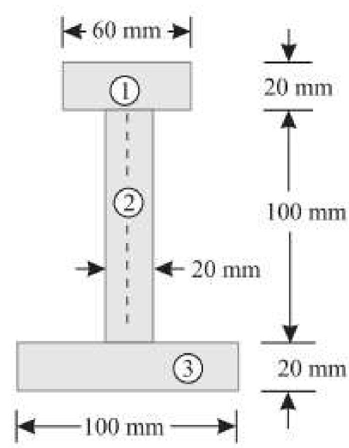
$$= \frac{150\,(50)^3}{12} = 1.5625 \times 10^6 \text{ mm}^4$$

Now moment of inertia of the whole section about *Y-Y* axis,

$$I_{YY} = (14.0625 \times 10^6) + (1.5625 \times 10^6) = 15.625 \times 10^6 \text{ mm}^4 \quad \textbf{Ans.}$$

EXAMPLE: An I-section is made up of three rectangles as shown in Fig. Find the moment of inertia of the section about the horizontal axis passing through the centre of gravity of the section.

Solution. First of all, let us find out centre of gravity of the section. As the section is symmetrical about *Y-Y* axis, therefore its centre of gravity will lie on this axis. Split up the whole section into three rectangles 1, 2 and 3 as shown in Fig. , Let bottom face of the bottom flange be the axis of reference.



(*i*) *Rectangle* 1

$$a_1 = 60 \times 20 = 1200 \text{ mm}$$

and $$y_1 = 20 + 100 + \frac{20}{2} = 130 \text{ mm}$$

(*ii*) *Rectangle* 2

$$a_2 = 100 \times 20 = 2000 \text{ mm}^2$$

and $$y_2 = 20 + \frac{100}{2} = 70 \text{ mm}$$

(*iii*) *Rectangle* 3

$$a_3 = 100 \times 20 = 2000 \text{ mm}^2$$

and $$y_3 = \frac{20}{2} = 10 \text{ mm}$$

We know that the distance between centre of gravity of the section and bottom face,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{(1200 \times 130) + (2000 \times 70) + (2000 \times 10)}{1200 + 2000 + 2000} \text{ mm}$$
$$= 60.8 \text{ mm}$$

We know that moment of inertia of rectangle (1) about an axis through its centre of gravity and parallel to *X-X* axis,

$$I_{G1} = \frac{60 \times (20)^3}{12} = 40 \times 10^3 \text{ mm}^4$$

and distance between centre of gravity of rectangle (1) and *X-X* axis,

$$h_1 = 130 - 60.8 = 69.2 \text{ mm}$$

$\therefore$ Moment of inertia of rectangle (1) about *X-X* axis,

$$= I_{G1} + a_1 h_1^2 = (40 \times 10^3) + [1200 \times (69.2)^2] = 5786 \times 10^3 \text{ mm}^4$$

Similarly, moment of inertia of rectangle (2) about an axis through its centre of gravity and parallel to *X-X* axis,

$$I_{G2} = \frac{20 \times (100)^3}{12} = 1666.7 \times 10^3 \text{ mm}^4$$

and distance between centre of gravity of rectangle (2) and *X-X* axis,

$$h_2 = 70 - 60.8 = 9.2 \text{ mm}$$

$\therefore$ Moment of inertia of rectangle (2) about *X-X* axis,

$$= I_{G2} + a_2 h_2^2 = (1666.7 \times 10^3) + [2000 \times (9.2)^2] = 1836 \times 10^3 \text{ mm}^4$$

Now moment of inertia of rectangle (3) about an axis through its centre of gravity and parallel to *X-X* axis,

$$I_{G3} = \frac{100 \times (20)^3}{12} = 66.7 \times 10^3 \text{ mm}^4$$

and distance between centre of gravity of rectangle (3) and *X-X* axis,

$$h_3 = 60.8 - 10 = 50.8 \text{ mm}$$

$\therefore$ Moment of inertia of rectangle (3) about *X-X* axis,

$$= I_{G3} + a_3 h_3^2 = (66.7 \times 10^3) + [2000 \times (50.8)^2] = 5228 \times 10^3 \text{ mm}^4$$

Now moment of inertia of the whole section about *X-X* axis,

$$I_{XX} = (5786 \times 10^3) + (1836 \times 10^3) + (5228 \times 10^3) = 12\,850 \times 10^3 \text{ mm}^4$$ **Ans.**