6. DYNAMICS

KINETICS: It is the branch of Dynamics, which deals with the bodies in motion due to the application of forces.

KINEMATICS: It is that branch of Dynamics, which deals with the bodies in motion, without any reference to the forces which are responsible for the motion.

PRINCIPLE OF DYNAMICS:

1. A body can posses acceleration only when some force is applied on it. Or in other words, if no force is applied on the body, then there will be no acceleration, and the body will continue to move with the existing uniform velocity.
2. The force applied on a body is proportional to the product of the mass of the body and the acceleration produced in it.

NEWTON'S LAWS OF MOTION:

Following are the three laws of motion, which were enunciated by Newton,

1. Newton's First Law of Motion states, "Everybody continues in its state of rest or of uniform motion, in a straight line, unless it is acted upon by some external force."

2. Newton's Second Law of Motion states, "The rate of change of momentum is directly proportional to the impressed force, and takes place in the same direction, in which the force acts."

$$\mathbf{F = ma = Mass \times Acceleration}$$

3. Newton's Third Law of Motion states, "To every action, there is always an equal and opposite reaction."

D'ALEMBERT'S PRINCIPLE:

It states, "If a rigid body is acted upon by a system of forces, this system may be reduced to a single resultant force whose magnitude, direction and the line of action may be found out by the methods of graphic statics."

We know that force acting on a body.

$$P = ma \qquad ...(i)$$

The equation (i) may also be written as :

$$P - ma = 0 \qquad ...(ii)$$

It may be noted that equation (i) is the equation of dynamics whereas the equation (ii) is the equation of statics. The equation (ii) is also known as the equation of dynamic equilibrium under the action of the real force P. This principle is known as D' Alembert's principle.

EQUATIONS OF MOTION:

Let u = Initial velocity,
v = Final velocity,
t = Time (in seconds) taken by the particle to change its velocity from u to v.
a = Uniform positive acceleration, and
s = Distance travelled in t seconds.

Since in t seconds, the velocity of the particle has increased steadily from (u) to (v) at the rate of a, therefore total increase in velocity = a t

$$v = u + a t \quad \ldots(i)$$

We know that distance travelled by the particle,
s = Average velocity × Time

$$= \left(\frac{u+v}{2}\right) \times t \quad \ldots(ii)$$

$$s = \left(\frac{u+u+at}{2}\right) \times t = ut + \frac{1}{2}at^2 \quad \ldots(iii)$$

$$v^2 = u^2 + 2as$$

EXAMPLE:

A scooter starts from rest and moves with a constant acceleration of 1.2 m/s2. Determine its velocity, after it has travelled for 60 meters.

Solution. Given : Initial velocity $(u) = 0$ (because it starts from rest) Acceleration (a) = 1.2 m/s^2 and distance travelled (s) = 60 m.

Let v = Final velocity of the scooter.

We know that $v^2 = u^2 + 2as = (0)^2 + 2 \times 1.2 \times 60 = 144$

$$v = 12 \text{ m/s} = \frac{12 \times 3600}{1000} = 43.2 \text{ km.p.h.} \quad \textbf{Ans.}$$

EXAMPLE: A motor car takes 10 seconds to cover 30 meters and12 seconds to cover 42 meters. Find the uniform acceleration of the car and its velocity at the end of 15 seconds.

Solution. Given : When $t = 10$ seconds, $s = 30$ m and when $t = 12$ seconds, $s = 42$ m.

Uniform acceleration

Let u = Initial velocity of the car, and

a = Uniform acceleration.

We know that the distance travelled by the car in 10 seconds,

$$30 = ut + \frac{1}{2}at^2 = u \times 10 + \frac{1}{2} \times a(10)^2 = 10u + 50\,a$$

Multiplying the above equation by 6,

$$180 = 60u + 300a \qquad ...(i)$$

Similarly, distance travelled by the car in 12 seconds,

$$42 = u \times 12 + \frac{1}{2} \times a(12)^2 = 12u + 72a$$

Mulitiplying the above equation by 5,

$$210 = 60u + 360a \qquad ...(ii)$$

Subtracting equation (i) from (ii),

$$30 = 60a \qquad \text{or} \qquad a = \frac{30}{60} = 0.5 \text{ m/s}^2 \quad \textbf{Ans.}$$

Velocity at the end of 15 seconds

Substituting the value of a in equation (i)

$$180 = 60u + (300 \times 0.5) = 60u + 150$$

$$\therefore \qquad u = \frac{(180 - 150)}{60} = 0.5 \text{ m/s}$$

We know that the velocity of the car after 15 seconds,

$$v = u + at = 0.5 + (0.5 \times 15) = 8 \text{ m/s} \quad \textbf{Ans.}$$

WORK: Whenever a force acts on a body, and the body undergoes some displacement, then work is said to be done. *e.g.*, if a force *P*, acting on a body, causes it to move through a distance *s* as shown in Fig.(*a*). Then work done by the force *P*

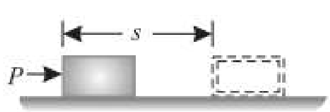
= Force × Distance

= *P* × *s*

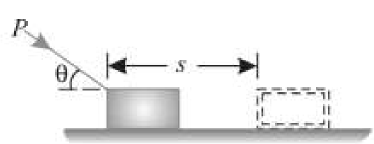
Sometimes, the force *P* does not act in the direction of motion of the body, or in other words, the body does not move in the direction of the force as shown in Fig.(*b*).
Then work done by the force *P*

= Component of the force in the direction of motion × Distance

= *P* cos θ × *s*



(*a*) Body moving in the direction of force



(*b*) Body not moving in the direction of force

UNITS OF WORK:

The units of work (or work done) are :
1**. One N-m**: It is the work done by a force of 1 N, when it displaces the body through 1 m. It is called joule (briefly written as J), Mathematically.

1 joule = 1 N-m

2. **One kN-m:** It is the work done by a force of 1 kN, when it displaces the body through 1 m. It is also called kilojoule (briefly written as kJ). Mathematically.

1 kilo-joule = 1 kN-m

POWER:
The power may be defined as the rate of doing work. It is thus the measure of performance of engines. e.g. an engine doing a certain amount of work, in one second, will be twice as powerful as an engine doing the same amount of work in two seconds.

UNITS OF POWER:
In S.I. units, the unit of power is watt (briefly written as W) which is equal to 1 N-m/s or 1 J/s. Generally, a bigger unit of power (kW) is used, which is equal to 10^3 W. Sometimes, a still bigger unit of power (MW) is also used, which is equal to 10^6 W.

ENERGY:
The energy may be defined as the capacity to do work. It exists in many forms i.e., mechanical, electrical chemical, heat, light etc. But in this subject, we shall deal in mechanical energy only.

UNITS OF ENERGY:
Since the energy of a body is measured by the work it can do, therefore the units of energy will be the same as those of the work.

POTENTIAL ENERGY:
It is the energy possessed by a body, for doing work, by virtue of its position. e.g.,
1. A body, raised to some height above the ground level, possesses some potential energy, because it can do some work by falling on the earth's surface.
2. Compressed air also possesses potential energy because it can do some work in expanding, to the volume it would occupy at atmospheric pressure.
3. A compressed spring also possesses potential energy, because it can do some work in recovering to its original shape.

Now consider a body of mass (m) raised through a height (h) above the datum level.
We know that work done in raising the body
= Weight × Distance = (mg) h = mgh
This work (equal to m.g.h) is stored in the body as potential energy.

KINETIC ENERGY:
It is the energy, possessed by a body, for doing work by virtue of its mass and velocity of motion.

$$KE = \frac{mv^2}{2}$$

LAW OF CONSERVATION OF ENERGY:
It states "The energy can neither be created nor destroyed, though it can be transformed from one form into any of the forms, in which the energy can exist."

From the above statement, it is clear, that no machine can either create or destroy energy, though it can only transform from one form into another. We know that the output of a machine is always less than the input of the machine. This is due to the reason that a part of the input is utilized in overcoming friction of the machine. This does not mean that this part of energy, which is used in overcoming the friction, has been destroyed. But it reappears in the form of heat energy at the bearings and other rubbing surfaces of the machine, though it is not available to us for useful work.

The above statement may be exemplified as below :
1. In an electrical heater, the electrical energy is converted into heat energy.
2. In an electric bulb, the electrical energy is converted into light energy.
3. In a dynamo, the mechanical energy is converted into electrical energy.

IMPULSE AND MOMENTUM:
Impulse is the change of momentum of an object when the object is acted upon by a force for an interval of time. So, with impulse, you can calculate the change in momentum, or you can use impulse to calculate the average impact force of a collision.

Impulse = Force X time

Momentum is the quantity of motion of a moving body, measured as a product of its mass and velocity.

Momentum = mass x velocity

PHENOMENON OF COLLISION:
Whenever two elastic bodies collide with each other, the phenomenon of collision takes place as given below :
1. The bodies, immediately after collision, come momentarily to rest.
2. The two bodies tend to compress each other, so long as they are compressed to the maximum value.
3. The two bodies attempt to regain its original shape due to their elasticity. This process of regaining the original shape is called **restitution**.
The time taken by the two bodies in compression, after the instant of collision, is called the time of compression and time for which restitution takes place is called the time of restitution. The sum of the two times of collision and restitution is called time of collision, period of collision, or period of impact.

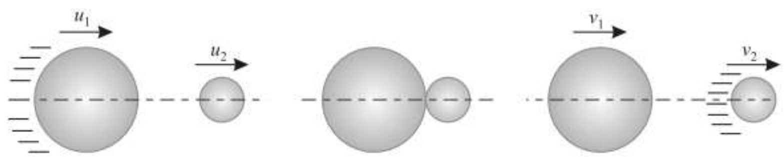
LAW OF CONSERVATION OF MOMENTUM:

It states, "The total momentum of two bodies remains constant after their collision or any other mutual action." Mathematically

$$\mathbf{m_1u_1 + m_2u_2 = m_1 v_1 + m_2 v_2}$$

Where; m_1 = Mass of the first body,
u_1 = Initial velocity of the first body,
v_1 = Final velocity of the first body, and
m_2, u_2, v_2 = Corresponding values for the second body.

COEFFICIENT OF RESTITUTION:



(a) Before impact (b) During impact (c) After impact

Consider two bodies A and B having a direct impact as shown in Fig. (a).

Let u1 = Initial velocity of the first body,
v1 = Final velocity of the first body, and
u2, v2 = Corresponding values for the second body.

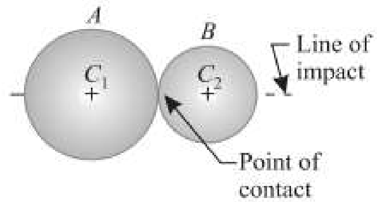
The impact will take place only if u1 is greater than u2.
Therefore, the velocity of approach will be equal to (u1 – u2). After impact, the separation of the two bodies will take place, only if v2 is greater than v1. Therefore the velocity of separation will be equal to (v2 – v1).
Now as per Newton's Law of Collision of Elastic Bodies:
Velocity of separation = e × Velocity of approach
(v2 – v1) = e (u1 – u2)
where e is a constant of proportionality, and is called the **coefficient of restitution**.

- Its value lies between 0 and 1. It may be noted that if e = 0, the two bodies are inelastic. But if e = 1, the two bodies are perfectly elastic.

NOTE:

- If the two bodies are moving in the same direction, before or after impact, then the velocity of approach or separation is the difference of their velocities. But if the two bodies are moving in the opposite directions, then the velocity of approach or separation is the algebraic sum of their velocities.

DIRECT COLLISION OF TWO BODIES:



The line of impact, of the two colliding bodies, is the line joining the centres of these bodies and passes through the point of contact or point of collision as shown in Fig. If the two bodies, before impact, are moving along the line of impact, the collision is called as direct impact as shown in Fig.

Now; $\mathbf{m_1u_1 + m_2u_2 = m_1 v_1 + m_2 v_2}$

NOTES: 1. Since the velocity of a body is a vector quantity, therefore its direction should always be kept in view while solving the examples.

2. If velocity of a body is taken as + ve in one direction, then the velocity in opposite direction should be taken as – ve.

3. If one of the bodies is initially at rest, then such a collision is also called impact.

EXAMPLE: A ball of mass 1 kg moving with a velocity of 2 m/s impinges directly on a ball of mass 2 kg at rest. The first ball, after impinging, comes to rest. Find the velocity of the second ball after the impact and the coefficient of restitution.

Solution. Given : Mass of first ball $(m_1) = 1$ kg ; Initial velocity of first ball $(u_1) = 2$ m/s ; Mass of second ball $(m_2) = 2$ kg ; Initial velocity of second ball $(u_2) = 0$ (because it is at rest) and final velocity of first ball after impact $(v_1) = 0$ (because, it comes to rest)

Velocity of the second ball after impact.

Let v_2 = Velocity of the second ball after impact.

We know from the law of conservation of momentum that

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$(1 \times 2) + (2 \times 0) = (1 \times 0) + (2 \times v_2)$$
$$\therefore \quad 2 = 2v_2$$
$$\text{or} \quad v_2 = 1 \text{ m/s} \quad \textbf{Ans.}$$

Coefficient of restitution

Let e = Coefficient of restitution.

We also know from the law of collision of elastic bodies that

$$(v_2 - v_1) = e\,(u_1 - u_2)$$
$$(1 - 0) = e\,(2 - 0)$$
$$\text{or} \quad e = \frac{1}{2} = 0.5 \quad \textbf{Ans.}$$

EXAMPLE: The masses of two balls are in the ratio of 2: 1 and their velocities are in the ratio of 1: 2, but in the opposite direction before impact. If the coefficient of restitution be 5/6, prove that after the impact, each ball will move back with 5/6th of its original velocity.

Solution. Given : Mass of first ball (m_1) = 2 M ; Mass of second ball (M_2) = M ; Initial velocity of first ball (u_1) = U ; Initial velocity of second ball (u_2) = $-2U$ (Minus sign due to opposite direction) and coefficient of restitution (e) = $\frac{5}{6}$

Let v_1 = Final velocity of the first ball, and

v_2 = Final velocity of the second ball.

We know from the law of conservation of momentum that

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$2M \times U + M(-2U) = 2Mv_1 + Mv_2$$

or $$0 = 2Mv_1 + Mv_2$$

$\therefore$ $$v_2 = -2v_1 \qquad \text{...}(i)$$

We also know from the law of collision of elastic bodies that

$$(v_2 - v_1) = e\,(u_1 - u_2) = \frac{5}{6}\left[U - (-2U)\right] = \frac{5U}{2} \qquad \text{...}(ii)$$

Substituting the value of v_2 from equation (i)

$$\left[-2v_1 - (v_1)\right] = \frac{5U}{2} \qquad \text{or} \qquad v_1 = -\frac{5}{6} \times U$$

Minus sign indicates that the direction of v_1 is opposite to that of U. Thus the first ball will move back with $\frac{5}{6}$th of its original velocity. **Ans.**

Now substituting the value of v_1 in equation (i),

$$v_2 = -2\left(-\frac{5}{6} \times U\right) = +\frac{5}{6} \times 2U$$

Plus sign indicates that the direction of v_2 is the same as that of v_1 or opposite to that of u_2. Thus the second ball will also move back with $\frac{5}{6}$th of its original velocity. **Ans.**