2. EQUILIBRIUM

EQUILIBRIUM:

If the resultant of a number of forces, acting on a particle is zero, the particle will be in equilibrium. Such a set of forces, whose resultant is zero, are called equilibrium forces. The force, which brings the set of forces in equilibrium is called an equilibrant.

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PRINCIPLES OF EQUILIBRIUM:

Though there are many principles of equilibrium, yet the following three are important from the subject point of view :

1. **Two force principle:**

As per this principle, if a body in equilibrium is acted upon by two forces, then they must be equal, opposite and collinear.

2. **Three force principle**:

As per this principle, if a body in equilibrium is acted upon by three forces, then the resultant of any two forces must be equal, opposite and collinear with the third force.

3. **Four force principle:**

As per this principle, if a body in equilibrium is acted upon by four forces, then the resultant of any two forces must be equal, opposite and collinear with the resultant of the other two forces.

METHODS FOR THE EQUILIBRIUM OF COPLANAR FORCES:

Though there are many methods of studying the equilibrium of forces, yet the following are important from the subject point of view :

1. Analytical method. 2. Graphical method.

LAMI'S THEOREM:

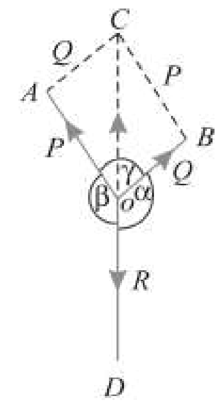
It states, "If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two."

Mathematically,

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

Proof

Consider three coplanar forces P, Q, and R acting at a point O. Let the opposite angles to three forces be α, β and γ as shown in Fig. 5.2.



Now let us complete the parallelogram $OACB$ with OA and OB as adjacent sides as shown in the figure. We know that the resultant of two forces P and Q will be given by the diagonal OC both in magnitude and direction of the parallelogram $OACB$.

Since these forces are in equilibrium, therefore the resultant of the forces P and Q must be in line with OD and equal to R, but in opposite direction.

From the geometry of the figure, we find

$$BC = P \text{ and } AC = Q$$

$$\therefore \quad \angle AOC = (180° - \beta)$$

and $$\angle ACO = \angle BOC = (180° - \alpha)$$

$$\therefore \quad \angle CAO = 180° - (\angle AOC + \angle ACO)$$
$$= 180° - [(180° - \beta) + (180° - \alpha)]$$
$$= 180° - 180° + \beta - 180° + \alpha$$
$$= \alpha + \beta - 180°$$

But $$\alpha + \beta + \gamma = 360°$$

Subtracting 180° from both sides of the above equation,

$$(\alpha + \beta - 180°) + \gamma = 360° - 180° = 180°$$

or $$\angle CAO = 180° - \gamma$$

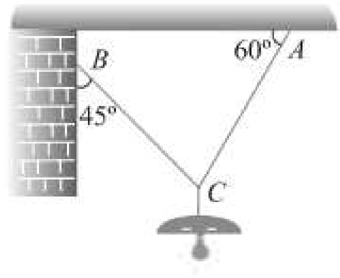
We know that in triangle AOC,

$$\frac{OA}{\sin \angle ACO} = \frac{AC}{\sin \angle AOC} = \frac{OC}{\sin \angle CAO}$$

$$\frac{OA}{\sin (180° - \alpha)} = \frac{AC}{\sin (180° - \beta)} = \frac{OC}{\sin (180° - \gamma)}$$

or $$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}$$

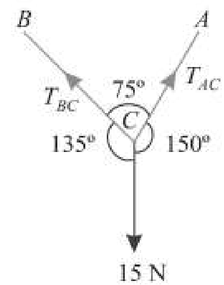
EXAMPLE: An electric light fixture weighting 15 N hangs from a point C, by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the horizontal as shown in Fig. Using Lami's theorem, or otherwise, determine the force in the strings AC.



Solution. Given : Weight at $C = 15$ N

Let T_{AC} = Force in the string AC, and

T_{BC} = Force in the string BC.



The system of forces is shown in Fig. 5.4. From the geometry of the figure, we find that angle between T_{AC} and 15 N is 150° and angle between T_{BC} and 15 N is 135°.

$\therefore \quad \angle ACB = 180° - (45° + 60°) = 75°$

Applying Lami's equation at C,

$$\frac{15}{\sin 75°} = \frac{T_{AC}}{\sin 135°} = \frac{T_{BC}}{\sin 150°}$$

or

$$\frac{15}{\sin 75°} = \frac{T_{AC}}{\sin 45°} = \frac{T_{BC}}{\sin 30°}$$

$$\therefore \quad T_{AC} = \frac{15 \sin 45°}{\sin 75°} = \frac{15 \times 0.707}{0.9659} = 10.98 \text{N} \quad \textbf{Ans.}$$

EXAMPLE: Two equal heavy spheres of 50 mm radius are in equilibrium within a smooth cup of 150 mm radius. Show that the reaction between the cup of one sphere is double than that between the two spheres.

Solution. Given : Radius of spheres = 50 mm and radius of the cup = 150 mm.

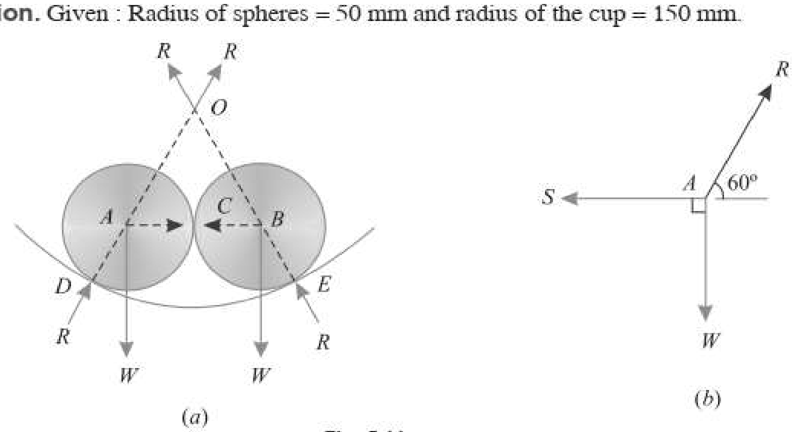


Fig. 5.11.

The two spheres with centres A and B, lying in equilibrium, in the cup with O as centre are shown in Fig. 5.11 (a). Let the two spheres touch each other at C and touch the cup at D and E respectively.

Let R = Reactions between the spheres and cup, and

S = Reaction between the two spheres at C.

From the geometry of the figure, we find that $OD = 150$ mm and $AD = 50$ mm. Therefore $OA = 100$ mm. Similarly $OB = 100$ mm. We also find that $AB = 100$ mm. Therefore OAB is an equilateral triangle. The system of forces at A is shown in Fig. 5.11 (*b*).

Applying Lami's equation at A,

$$\frac{R}{\sin 90°} = \frac{W}{\sin 120°} = \frac{S}{\sin 150°}$$

$$\frac{R}{1} = \frac{W}{\sin 60°} = \frac{S}{\sin 30°}$$

$$\therefore \quad R = \frac{S}{\sin 30°} = \frac{S}{0.5} = 2S$$

Hence the reaction between the cup and the sphere is double than that between the two spheres. **Ans.**

GRAPHICAL METHOD FOR THE EQUILIBRIUM OF COPLANAR FORCES:

We have studied that the equilibrium of forces by analytical method. Sometimes, the analytical method is too tedious and complicated. The equilibrium of such forces may also be studied, graphically, by drawing the vector diagram. This may also be done by studying the

1. Converse of the Law of Triangle of Forces
2. Converse of the Law of Polygon of Forces

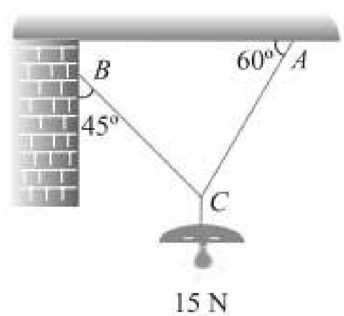
CONVERSE OF THE LAW OF TRIANGLE OF FORCES:

If three forces acting at a point be represented in magnitude and direction by the three sides a triangle, taken in order, the forces shall be in equilibrium.

CONVERSE OF THE LAW† OF POLYGON OF FORCES:

If any number of forces acting at a point be represented in magnitude and direction by the sides of a closed polygon, taken in order, the forces shall be in equilibrium.

EXAMPLE: An electric light fixture weighing 15 N hangs from a point C, by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the horizontal as shown in Fig.

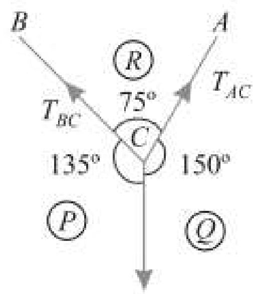


SOLUTION: Given. Weight at *C* = 15 N

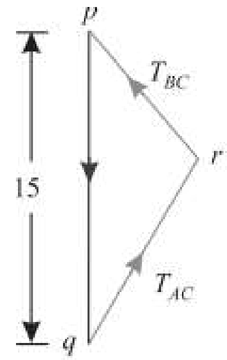
TAC = Force in the string *AC*, and

TBC = Force in the string *BC*.

First of all, draw the space diagram for the joint *C* and name the forces according to Bow's notations as shown in Fig. The force *TAC* is named as *RQ* and the force *TBC* as *PR*.



(*a*) Space diagram



(*a*) Vector diagram

Now draw the vector diagram for the given system of forces as shown in Fig. (*b*) and as discussed below;

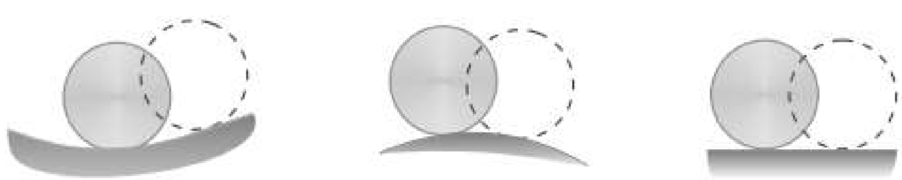
- Select some suitable point p and draw a vertical line pq equal to 15 N to some suitable scale representing weight (PQ) of the electric fixture**.**
- Through p draw a line pr parallel to PR and through q, draw a line qr parallel to QR. Let these two lines meet at r and close the triangle pqr, which means that joint C is in equilibrium.
- By measurement, we find that the forces in strings AC (TAC) and BC (TPC) is equal to 1.0 N and 7.8 N respectively.

CONDITIONS OF EQUILIBRIUM: If the body is completely at rest, it necessarily means that there is neither a resultant force nor a couple acting on it. A little consideration will show, that in this case the following conditions are already satisfied:

$$\sum H = 0 \quad \sum V = 0 \text{ and } \sum M = 0$$

The above mentioned three equations are known as the conditions of equilibrium.

TYPES OF EQUILIBRIUM:



(*a*) Stable (b) Unstable (*c*) Neutral

1. **STABLE EQUILIBRIUM:** A body is said to be in stable equilibrium, if it returns back to its original position, after it is slightly displaced from its position of rest. This happens when some additional force sets up due to displacement and brings the body back to its original position. A smooth cylinder, lying in a curved surface, is in stable equilibrium. If we slightly displace the cylinder from its position of rest (as shown by dotted lines), it will tend to return back to its original position in order to bring its weight normal to horizontal axis as shown in Fig. (*a*).

2. **UNSTABLE EQUILIBRIUM:** A body is said to be in an unstable equilibrium, if it does not return back to its original position, and heels farther away, after slightly displaced from its position of rest. This happens when the additional force moves the body away from its position of rest. This happens when the additional force moves the body away from its position of rest. A smooth cylinder lying on a convex surface is in unstable equilibrium. If we slightly displace the cylinder from its position of rest (as shown by dotted lines) the body will tend to move away from its original position as shown in Fig. (*b*).

3. **NEUTRAL EQUILIBRIUM:** A body is said to be in a neutral equilibrium, if it occupies a new position (and remains at rest in this position) after slightly displaced from its position of rest. This happens when no additional force sets up due to the displacement. A smooth cylinder lying on a horizontal plane is in neutral equilibrium as shown in Fig. (*c*).