3. FRICTION

INTRODUCTION:

If a block of one substance is placed over the level surface of the same or different material, a certain degree of interlocking of the minutely projecting particles takes place. This does not involve any force, so long as the block does not move or tends to move. But whenever one of the blocks moves or tends to move tangentially with respect to the surface, on which it rests, the interlocking property of the projecting particles opposes the motion. This opposing force, which acts in the opposite direction of the movement of the block, is called *force of friction* or simply *friction.* It is of the following two types:

1. Static friction. **2.** Dynamic friction

STATIC FRICTION:

It is the friction experienced by a body when it is at rest. Or in other words, it is the friction when the body tends to move.

DYNAMIC FRICTION:

It is the friction experienced by a body when it is in motion. It is also called kinetic friction. The dynamic friction is of the following two types:

1. **Sliding friction**: It is the friction, experienced by a body when it slides over another body.
2. **Rolling friction**: It is the friction, experienced by a body when it rolls over another body.

LIMITING FRICTION: The maximum value of frictional force, which comes into play, when a body just begins to slide over the surface of the other body, is known as limiting friction. It may be noted that when the applied force is less than the limiting friction, the body remains at rest, and the friction is called static friction, which may have any value between zero and limiting friction.

COEFFICIENT OF FRICTION:

It is the ratio of limiting friction to the normal reaction, between the two bodies, and is generally denoted by μ.

Mathematically, coefficient of friction,

$$\mu = \frac{F}{R} = \tan \phi \qquad \text{or} \qquad F = \mu R$$

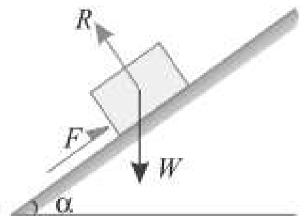
ϕ = Angle of friction,

F = Limiting friction, and

R = Normal reaction between the two bodies.

ANGLE OF FRICTION: Consider a body of weight *W* resting on an inclined plane as shown in Fig. We know that the body is in equilibrium under the action of the following forces:

1. Weight (*W*) of the body, acting vertically downwards,
2. Friction force (*F*) acting upwards along the plane, and
3. Normal reaction (*R*) acting at right angles to the plane.



Let the angle of inclination (α) be gradually increased, till the body just starts sliding down the plane. This angle of inclined plane, at which a body just begins to slide down the plane, is called the angle of friction. This is also equal to the angle, which the normal reaction makes with the vertical.

ANGLE OF REPOSE: Angle of repose is defined as the angle of the inclined plane with horizontal such that a body placed on it just begins to slide.

LAWS OF FRICTION:

Prof. Coulomb, after extensive experiments, gave some laws of friction, which may be grouped under the following heads :

1. Laws of static friction, and
2. Laws of kinetic or dynamic friction

LAWS OF STATIC FRICTION:

Following are the laws of static friction :

1. The force of friction always acts in a direction, opposite to that in which the body tends to move, if the force of friction would have been absent.
2. The magnitude of the force of friction is exactly equal to the force, which tends to move the body.
3. The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two surfaces. Mathematically:

$$F/R = CONSTANT$$

4. The force of friction is independent of the area of contact between the two surfaces.
5. The force of friction depends upon the roughness of the surfaces

LAWS OF KINETIC OR DYNAMIC FRICTION:

Following are the laws of kinetic or dynamic friction:

1. The force of friction always acts in a direction, opposite to that in which the body is moving.
2. The magnitude of kinetic friction bears a constant ratio to the normal reaction between the two surfaces. But this ratio is slightly less than that in case of limiting friction.
3. For moderate speeds, the force of friction remains constant. But it decreases slightly with the increase of speed.

ADVANTAGES OF FRICTION:

- Friction is responsible for many types of motion
- It helps us walk on the ground
- Brakes in a car make use of friction to stop the car

- Asteroids are burnt in the atmosphere before reaching Earth due to friction.
- It helps in the generation of heat when we rub our hands.

DISADVANTAGES OF FRICTION:

- Friction produces unnecessary heat leading to the wastage of energy.
- The force of friction acts in the opposite direction of motion, so friction slows down the motion of moving objects.
- A lot of money goes into preventing friction and the usual wear and tear caused by it by using techniques like greasing and oiling.

EQUILIBRIUM OF A BODY ON A ROUGH HORIZONTAL PLANE:

We know that a body, lying on a rough horizontal plane will remain in equilibrium. But whenever a force is applied on it, the body will tend to move in the direction of the force. In such cases, equilibrium of the body is studied first by resolving the forces horizontally and then vertically.
Now the value of the force of friction is obtained from the relation:

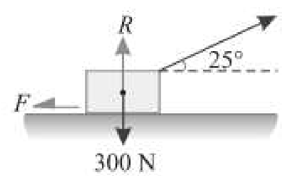
F = Mr

EXAMPLE: A body of weight 300 N is lying on a rough horizontal plane having a coefficient of friction as 0.3. Find the magnitude of the force, which can move the body, while acting at an angle of 25° with the horizontal.

Solution. Given: Weight of the body (W) = 300 N; Coefficient of friction (μ) = 0.3 and angle made by the force with the horizontal (α) = 25°

Let P = Magnitude of the force, which can move the body, and

F = Force of friction.



Resolving the forces horizontally,

$$F = P \cos \alpha = P \cos 25° = P \times 0.9063$$

and now resolving the forces vertically,

$$R = W - P \sin \alpha = 300 - P \sin 25°$$
$$= 300 - P \times 0.4226$$

We know that the force of friction (F),

$$0.9063\, P = \mu R = 0.3 \times (300 - 0.4226\, P) = 90 - 0.1268\, P$$

or $$90 = 0.9063\, P + 0.1268\, P = 1.0331\, P$$

$$\therefore \quad P = \frac{90}{1.0331} = 87.1 \text{ N} \quad \textbf{Ans.}$$

EQUILIBRIUM OF A BODY ON A ROUGH INCLINED PLANE:
Consider a body, of weight *W*, lying on a rough plane inclined at an angle α with the horizontal as shown in Fig.(*a*) and (*b*).

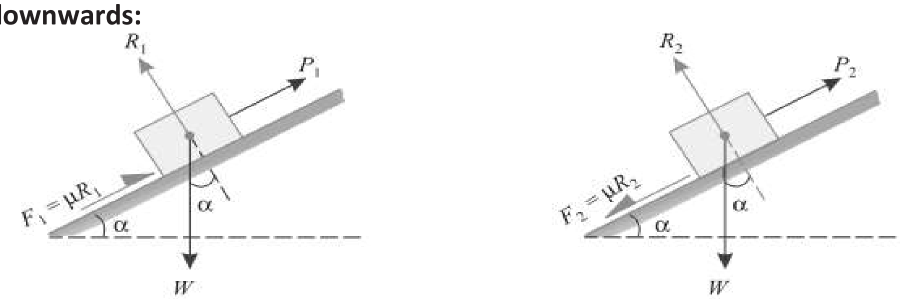
(*a*) Angle of inclination less than the angle of friction

(*b*) Angle of inclination more than the angle of friction

A little consideration will show, that if the inclination of the plane, with the horizontal, is less the angle of friction, the body will be automatically in equilibrium as shown in Fig. (*a*). If in this condition, the body is required to be moved upwards or downwards, a corresponding force is required, for the same. But, if the inclination of the plane is more than the angle of friction, the body will move down. And an upward force (*P*) will be required to resist the body from moving down the plane as shown in Fig. (b).
Though there are many types of forces, for the movement of the body, yet the following are important from the subject point of view :
1. Force acting along the inclined plane.
2. Force acting horizontally.
3. Force acting at some angle with the inclined plane.

EQUILIBRIUM OF A BODY ON A ROUGH INCLINED PLANE SUBJECTED TO A FORCE ACTING ALONG THE INCLINED PLANE:
Consider a body lying on a rough inclined plane subjected force acting along the inclined plane, which keeps it in equilibrium as shown in Fig.(*a*) and (*b*).
Let *W* = Weight of the body,
α = Angle, which the inclined plane makes with the horizontal,
R = Normal reaction,
μ = Coefficient of friction between the body and the inclined plane, and
ϕ = Angle of friction, such that μ = tan ϕ.
A little consideration will show that if the force is not there, the body will slide down the plane. Now we shall discuss the following two cases:

1. Minimum force (P1) which will keep the body in equilibrium, when it is at the point of sliding downwards:



(*a*) Body at the point of sliding downwards

(*b*) Body at the point of sliding upwards

In this case, the force of friction ($F_1 = \mu.R_1$) will act upwards, as the body is at the point of sliding downwards as shown in Fig. 8.8 (*a*). Now resolving the forces along the plane,

$$P_1 = W \sin \alpha - \mu.R_1 \qquad ...(i)$$

and now resolving the forces perpendicular to the plane.

$$R_1 = W \cos \alpha \qquad ...(ii)$$

Substituting the value of R_1 in equation (*i*).

$$P_1 = W \sin \alpha - \mu\, W \cos \alpha = W (\sin \alpha - \mu \cos \alpha)$$

and now substituting the value of $\mu = \tan \phi$ in the above equation,

$$P_1 = W (\sin \alpha - \tan \phi \cos \alpha)$$

Multiplying both sides of this equation by $\cos \phi$,

$$P_1 \cos \phi = W (\sin \alpha \cos \phi - \sin \phi \cos \alpha) = W \sin (\alpha - \phi)$$

$$\therefore \qquad P_1 = W \times \frac{\sin (\alpha - \phi)}{\cos \phi}$$

2. *Maximum force (P_2) which will keep the body in equilibrium, when it is at the point of sliding upwards.*

In this case, the force of friction ($F_2 = \mu.R_2$) will act downwards as the body is at the point of sliding upwards as shown in Fig. 8.8 (*b*). Now resolving the forces along the plane,

$$P_2 = W \sin \alpha + \mu.R_2 \qquad ...(i)$$

and now resolving the forces perpendicular to the plane,

$$R_2 = W \cos \alpha \qquad ...(ii)$$

Substituting the value of R_2 in equation (*i*),

$$P_2 = W \sin \alpha + \mu\, W \cos \alpha = W (\sin \alpha + \mu \cos \alpha)$$

and now substituting the value of $\mu = \tan \phi$ in the above equation,

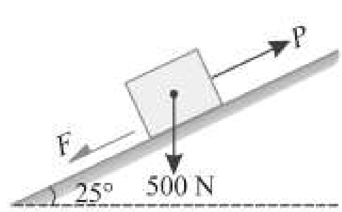
$$P_2 = W (\sin \alpha + \tan \phi \cos \alpha)$$

Multiplying both sides of this equation by $\cos \phi$,

$$P_2 \cos \phi = W (\sin \alpha \cos \phi + \sin \phi \cos \alpha) = W \sin (\alpha + \phi)$$

$$\therefore \qquad P_2 = W \times \frac{\sin (\alpha + \phi)}{\cos \phi}$$

EXAMPLE: A body of weight 500 N is lying on a rough plane inclined at an angle of 25° with the horizontal. It is supported by an effort (P) parallel to the plane as shown in Fig. Determine the minimum and maximum values of P, for which the equilibrium can exist, if the angle of friction is 20°.



Solution. Given: Weight of the body (W) = 500 N ; Angle at which plane is inclined (α) = 25° and angle of friction (ϕ) = 20°.

Minimum value of P

We know that for the minimum value of P, the body is at the point of sliding downwards. We also know that when the body is at the point of sliding downwards, then the force

$$P_1 = W \times \frac{\sin(\alpha - \phi)}{\cos \phi} = 500 \times \frac{\sin(25° - 20°)}{\cos 20°} \text{ N}$$

$$= 500 \times \frac{\sin 5°}{\cos 20°} = 500 \times \frac{0.0872}{0.9397} = 46.4 \text{ N} \qquad \textbf{Ans.}$$

Maximum value of P

We know that for the maximum value of P, the body is at the point of sliding upwards. We also know that when the body is at the point of sliding upwards, then the force

$$P_2 = W \times \frac{\sin(\alpha + \phi)}{\cos \phi} = 500 \times \frac{\sin(25° + 20°)}{\cos 20°} \text{ N}$$

$$= 500 \times \frac{\sin 45°}{\cos 20°} = 500 \times \frac{0.7071}{0.9397} = 376.2 \text{ N} \qquad \textbf{Ans.}$$

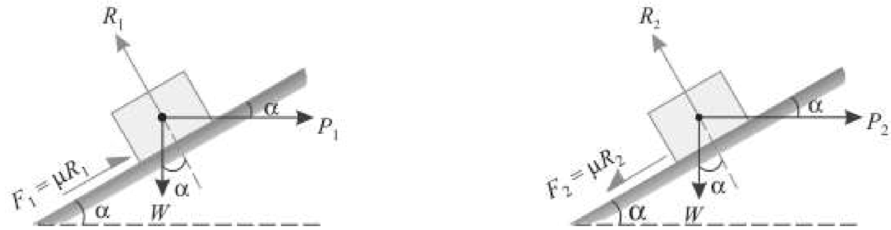
EQUILIBRIUM OF A BODY ON A ROUGH INCLINED PLANE SUBJECTED TO A FORCE ACTING HORIZONTALLY:

Consider a body lying on a rough inclined plane subjected to a force acting horizontally, which keeps it in equilibrium as shown in Fig.(*a*) and (*b*).

W = Weight of the body,
α = Angle, which the inclined plane makes with the horizontal,
R = Normal reaction,
μ = Coefficient of friction between the body and the inclined plane, and
ϕ = Angle of friction, such that $\mu = \tan \phi$.

A little consideration will show that if the force is not there, the body will slide down on the plane.
Now we shall discuss the following two cases:

1. *Minimum force (P_1) which will keep the body in equilibrium, when it is at the point of sliding downwards.*



(*a*) Body at the point of sliding downwards

(*b*) Body at the point of sliding upwards

Fig. 8.13.

In this case, the force of friction ($F_1 = \mu . R_1$) will act upwards, as the body is at the point of sliding downwards as shown in Fig. 8.13. (*a*). Now resolving the forces along the plane,

$$P_1 \cos \alpha = W \sin \alpha - \mu R_1 \quad ...(i)$$

and now resolving the forces perpendicular to the plane,

$$R_1 = W \cos \alpha + P_1 \sin \alpha \quad ...(ii)$$

Substituting this value of R_1 in equation (*i*),

$$P_1 \cos \alpha = W \sin \alpha - \mu(W \cos \alpha + P_1 \sin \alpha)$$

$$= W \sin \alpha - \mu W \cos \alpha - \mu P_1 \sin \alpha$$

$$P_1 \cos \alpha + \mu P_1 \sin \alpha = W \sin \alpha - \mu W \cos \alpha$$

$$P_1(\cos \alpha + \mu \sin \alpha) = W(\sin \alpha - \mu \cos \alpha)$$

$$\therefore \quad P_1 = W \times \frac{(\sin \alpha - \mu \cos \alpha)}{(\cos \alpha + \mu \sin \alpha)}$$

Now substituting the value of $\mu = \tan \phi$ in the above equation,

$$P_1 = W \times \frac{(\sin \alpha - \tan \phi \cos \alpha)}{(\cos \alpha + \tan \phi \sin \alpha)}$$

Multiplying the numerator and denominator by $\cos \phi$,

$$P_1 = W \times \frac{\sin \alpha \cos \phi - \sin \phi \cos \alpha}{\cos \alpha \cos \phi + \sin \alpha \sin \phi} = W \times \frac{\sin(\alpha - \phi)}{\cos(\alpha - \phi)}$$

$$= W \tan(\alpha - \phi) \quad ...(\text{when } \alpha > \phi)$$

$$= W \tan(\phi - \alpha) \quad ...(\text{when } \phi > \alpha)$$

2. *Maximum force* (P_2) *which will keep the body in equilibrium, when it is at the point of sliding upwards*

In this case, the force of friction ($F_2 = \mu R_2$) will act downwards, as the body is at the point of sliding upwards as shown in Fig.8.12. (*b*). Now resolving the forces along the plane,

$$P_2 \cos \alpha = W \sin \alpha + \mu R_2 \quad ...(iii)$$

and now resolving the forces perpendicular to the plane,

$$R_2 = W \cos \alpha + P_2 \sin \alpha \quad ...(iv)$$

Substituting this value of R_2 in the equation (*iii*),

$$P_2 \cos \alpha = W \sin \alpha + \mu (W \cos \alpha + P_2 \sin \alpha)$$
$$= W \sin \alpha + \mu W \cos \alpha + \mu P_2 \sin \alpha$$
$$P_2 \cos \alpha - \mu P_2 \sin \alpha = W \sin \alpha + \mu W \cos \alpha$$
$$P_2 (\cos \alpha - \mu \sin \alpha) = W (\sin \alpha + \mu \cos \alpha)$$
$$\therefore \quad P_2 = W \times \frac{(\sin \alpha + \mu \cos \alpha)}{(\cos \alpha - \mu \sin \alpha)}$$

Now substituting the value of $\mu = \tan \phi$ in the above equation,

$$P_2 = W \times \frac{\sin \alpha + \tan \phi \cos \alpha}{\cos \alpha - \tan \phi \sin \alpha}$$

Multiplying the numerator and denominator by $\cos \phi$,

$$P_2 = W \times \frac{\sin \alpha \cos \phi + \sin \phi \cos \alpha}{\cos \alpha \cos \phi - \sin \phi \sin \alpha} = W \times \frac{\sin (\alpha + \phi)}{\cos (\alpha + \phi)}$$
$$= W \tan (\alpha + \phi)$$

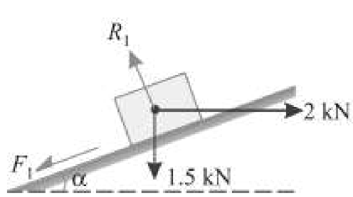
EXAMPLE: A load of 1.5 kN, resting on an inclined rough plane, can be moved up the plane by a force of 2 kN applied horizontally or by a force 1.25 kN applied parallel to the plane. Find the inclination of the plane and the coefficient of friction.

Solution. Given: Load (W) = 1.5 kN; Horizontal effort (P_1) = 2 kN and effort parallel to the inclined plane (P_2) = 1.25 kN.

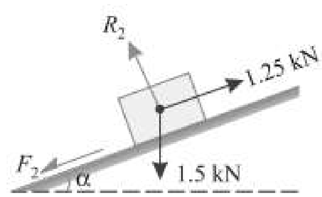
Inclination of the plane

Let α = Inclination of the plane, and

ϕ = Angle of friction.



(*a*) Horizontal force



(*b*) Force parallel to the plane

First of all, consider the load of 1.5 kN subjected to a horizontal force of 2 kN as shown in Fig. 8.14 (*a*). We know that when the force is applied horizontally, then the magnitude of the force, which can move the load up the plane.

$$P = W \tan(\alpha + \phi)$$

or $$2 = 1.5 \tan(\alpha + \phi)$$

$$\therefore \quad \tan(\alpha + \phi) = \frac{2}{1.5} = 1.333 \quad \text{or} \quad (\alpha + \phi) = 53.1°$$

Now consider the load of 1.5 kN subjected to a force of 1.25 kN along the plane as shown in Fig. 8.14 (*b*). We Know that when the force is applied parallel to the plane, then the magnitude of the force, which can move the load up the plane,

$$P = W \times \frac{\sin(\alpha + \phi)}{\cos \phi}$$

or $$1.25 = 1.5 \times \frac{\sin 53.1°}{\cos \phi} = 1.5 \times \frac{0.8}{\cos \phi} = \frac{1.2}{\cos \phi}$$

$$\therefore \quad \cos \phi = \frac{1.2}{1.25} = 0.96 \quad \text{or} \quad \phi = 16.3°$$

and $$\alpha = 53.1° - 16.3° = 36.8° \quad \textbf{Ans.}$$

Coefficient of friction

We know that the coefficient of friction,

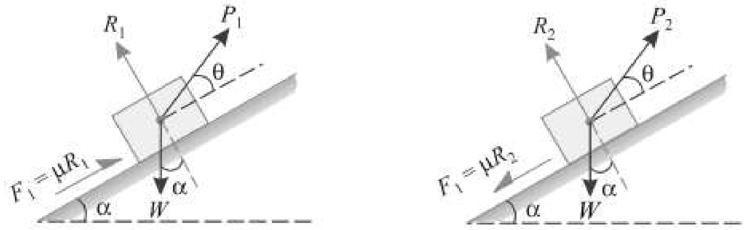
$$\mu = \tan \phi = \tan 16.3° = 0.292 \quad \textbf{Ans.}$$

EQUILIBRIUM OF A BODY ON A ROUGH INCLINED PLANE SUBJECTED TO A FORCE ACTING AT SOME ANGLE WITH THE INCLINED PLANE:

Consider a body lying on a rough inclined plane subjected to a force acting at some angle with the inclined plane, which keeps it in equilibrium as shown in Fig.(*a*) and (*b*).

Let W = Weight of the body,
α = Angle which the inclined plane makes with the horizontal,
θ = Angle which the force makes with the inclined surface,
R = Normal reaction,
μ = Coefficient of friction between the body and the inclined plane, and
ϕ = Angle of friction, such that $\mu = \tan \phi$.
A little consideration will show that if the force is not there, the body will slide down the plane.
Now we shall discuss the following two cases :

1. *Minimum force (P_1) which will keep the body in equilibrium when it is at the point of sliding downwards.*



(*a*) Body at the point of sliding downwards

(*b*) Body at the point of sliding upwards

Fig. 8.17.

In this case, the force of friction ($F_1 = \mu R_1$) will act upwards, as the body is at the point of sliding downwards as shown in Fig. 8.17 (*a*). Now resolving the forces along the plane,

$$P_1 \cos \theta = W \sin \alpha - \mu R_1 \qquad ...(i)$$

and now resolving the forces perpendicular to the plane,

$$R_1 = W \cos \alpha - P_1 \sin \theta \qquad ...(ii)$$

Substituting the value of R_1 in equation (*i*),

$$P_1 \cos \theta = W \sin \alpha - \mu\, (W \cos \alpha - P_1 \sin \theta)$$

$$= W \sin \alpha - \mu\, W \cos \alpha + \mu\, P_1 \sin \theta$$

$$P_1 \cos \theta - \mu\, P_1 \sin \theta = W \sin \alpha - \mu\, W \cos \alpha$$

$$P_1 (\cos \theta - \mu \sin \theta) = W (\sin \alpha - \mu \cos \alpha)$$

$$\therefore \qquad P_1 = W \times \frac{(\sin \alpha - \mu \cos \alpha)}{(\cos \theta - \mu \sin \theta)}$$

and now substituting the value of $\mu = \tan \phi$ in the above equation,

$$P_1 = W \times \frac{(\sin \alpha - \tan \phi \cos \alpha)}{(\cos \theta - \tan \phi \sin \theta)}$$

Multiplying the numerator and denominator by $\cos \phi$,

$$P_1 = W \times \frac{(\sin \alpha \cos \phi - \sin \phi \cos \alpha)}{(\cos \theta \cos \phi - \sin \phi \sin \theta)} = W \times \frac{\sin (\alpha - \phi)}{\cos (\theta + \phi)}$$

2. *Maximum force* (P_2) *which will keep the body in equilibrium, when it is at the point of sliding upwards.*

In this case, the force of friction ($F_2 = \mu R_2$) will act downwards as the body is at the point of sliding upwards as shown in Fig. 8.17 (*b*). Now resolving the forces along the plane.

$$P_2 \cos \theta = W \sin \alpha + \mu R_2 \quad ...(iii)$$

and now resolving the forces perpendicular to the plane,

$$R_2 = W \cos \alpha - P_2 \sin \theta \quad ...(iv)$$

Substituting the value of R_2 in equation (*iii*),

$$P_2 \cos \theta = W \sin \alpha + \mu (W \cos \alpha - P_2 \sin \theta)$$

$$= W \sin \alpha + \mu W \cos \alpha - \mu P_2 \sin \theta$$

$$P_2 \cos \theta + \mu P_2 \sin \theta = W \sin \alpha + \mu W \cos \alpha$$

$$P_2 (\cos \theta + \mu \sin \theta) = W (\sin \alpha + \mu \cos \alpha)$$

$$\therefore \quad P_2 = W \times \frac{(\sin \alpha + \mu \cos \alpha)}{(\cos \theta + \mu \sin \theta)}$$

and now substituting the vaue of $\mu = \tan \phi$ in the above equation,

$$P_2 = W \times \frac{(\sin \alpha + \tan \phi \cos \alpha)}{(\cos \theta + \tan \phi \sin \theta)}$$

Multiplying the numerator and denominator by $\cos \phi$,

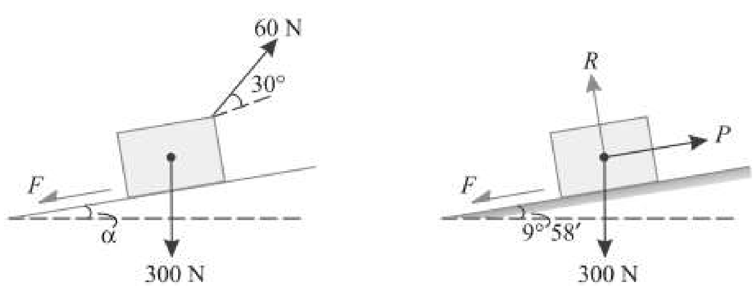
$$P_2 = W \times \frac{(\sin \alpha \cos \phi + \sin \phi \cos \alpha)}{(\cos \theta \cos \phi + \sin \phi \sin \theta)} = W \times \frac{\sin (\alpha + \phi)}{\cos (\theta - \phi)}$$

EXAMPLE: Find the force required to move a load of 300 N up a rough plane, the force being applied parallel to the plane. The inclination of the plane is such that when the same load is kept on a perfectly smooth plane inclined at the same angle, a force of 60 N applied at an inclination of 30° to the plane, keeps the same load in equilibrium. Assume coefficient of friction between the rough plane and the load to be equal to 0.3.

Solution. Given: Load (W) = 300 N; Force (P_1) = 60 N and angle at which force is inclined (θ) = 30°.

Let α = Angle of inclination of the plane.

First of all, consider the load lying on a smooth plane inclined at an angle (α) with the horizontal and subjected to a force of 60 N acting at an angle 30° with the plane as shown in Fig. 8.18 (*a*).



(*a*) Smooth plane (*b*) Rough plane

We know that in this case, because of the smooth plane $\mu = 0$ or $\phi = 0$. We also know that the force required, when the load is at the point of sliding upwards (P),

$$60 = W \times \frac{\sin(\alpha + \phi)}{\cos(\theta - \phi)} = 300 \times \frac{\sin \alpha}{\cos 30°} = 300 \times \frac{\sin \alpha}{0.866} = 346.4 \sin \alpha$$

$$...(\because \phi = 0)$$

or $$\sin \alpha = \frac{60}{346.4} = 0.1732 \quad \text{or} \quad \alpha = 10°$$

Now consider the load lying on the rough plane inclined at an angle of 10° with the horizontal as shown in Fig. 8.18. (*b*). We know that in this case, $\mu = 0.3 = \tan \phi$ or $\phi = 16.7°$.

We also know that force required to move the load up the plane,

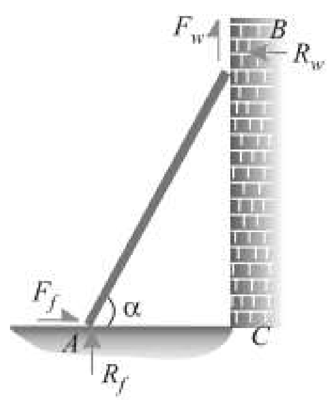
$$P = W \times \frac{\sin(\alpha + \phi)}{\cos \phi} = 300 \times \frac{\sin(10° + 16.7°)}{\cos 16.7°} \text{ N}$$

$$= 300 \times \frac{\sin 26.7°}{\cos 16.7°} = 300 \times \frac{0.4493}{0.9578} = 140.7 \text{ N} \quad \textbf{Ans.}$$

APPLICATIONS OF FRICTION

LADDER FRICTION: The ladder is a device for climbing or scaling on the roofs or walls. It consists of two long uprights of wood, iron or rope connected by a number of cross pieces called rungs. These runing serve as steps.

Consider a ladder *AB* resting on the rough ground and leaning against a wall, as shown in Fig.



As the upper end of the ladder tends to slip downwards, therefore the direction of the force of friction between the ladder and the wall (F_w) will be upwards as shown in the figure. Similarly, as the lower end of the ladder tends to slip away from the wall, therefore the direction of the force of friction between the ladder and the floor (F_f) will be towards the wall as shown in the figure.

Since the system is in equilibrium, therefore the algebraic sum of the horizontal and vertical components of the forces must also be equal to zero.

Note: The normal reaction at the floor (R_f) will act perpendicular of the floor. Similarly, normal reaction of the wall (R_w) will also act perpendicular to the wall.

EXAMPLE: A ladder 5 meters long rests on a horizontal ground and leans against a smooth vertical wall at an angle 70° with the horizontal. The weight of the ladder is 900 N and acts at its middle. The ladder is at the point of sliding, when a man weighing 750N stands on a rung 1.5 metre from the bottom of the ladder. Calculate the coefficient of friction between the ladder and the floor.

Solution. Given: Length of the ladder (l) = 5 m; Angle which the ladder makes with the horizontal (α) = 70°; Weight of the ladder (w_1) = 900 N; Weight of man (w_2) = 750 N and distance between the man and bottom of ladder = 1.5 m.

Forces acting on the ladder are shown in Fig. 9.3.

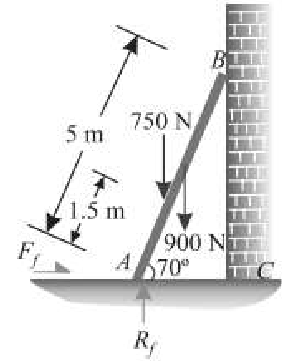


Fig. 9.3.

Let μ_f = Coefficient of friction between ladder and floor and

R_f = Normal reaction at the floor.

Resolving the forces vertically,

$$R_f = 900 + 750 = 1650 \text{ N} \quad ...(i)$$

$\therefore$ Force of friction at A

$$F_f = \mu_f \times R_f = \mu_f \times 1650 \quad ...(ii)$$

Now taking moments about B, and equating the same,

$$R_f \times 5 \sin 20° = (F_f \times 5 \cos 20°) + (900 \times 2.5 \sin 20°) + (750 \times 3.5 \sin 20°)$$

$$= (F_f \times 5 \cos 20°) + (4875 \sin 20°)$$

$$= (\mu_f \times 1650 \times 5 \cos 20°) + 4875 \sin 20°$$

and now substituting the values of R_f and F_f from equations (i) and (ii)

$$1650 \times 5 \sin 20° = (\mu_f \times 1650 \times 5 \cos 20°) + (4875 \sin 20°)$$

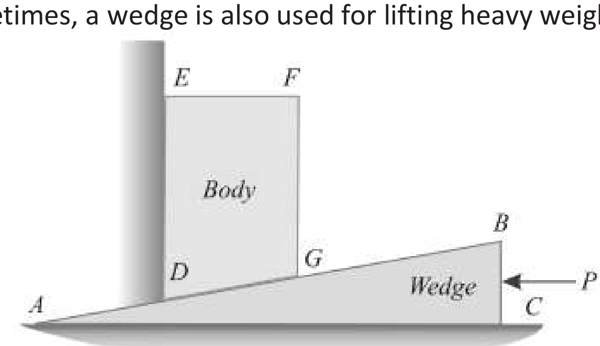
Dividing both sides by 5 sin 20°,

$$1650 = (\mu_f \times 1650 \cot 20°) + 975$$

$$= (\mu_f \times 1650 \times 2.7475) + 975 = 4533\,\mu_f + 975$$

$$\therefore \quad \mu_f = \frac{1650 - 975}{4533} = 0.15 \quad \textbf{Ans.}$$

WEDGE FRICTION: A wedge is, usually, of a triangular or trapezoidal in cross-section. It is, generally, used for slight adjustements in the position of a body *i.e.* for tightening fits or keys for shafts. Sometimes, a wedge is also used for lifting heavy weights as shown in Fig.



It will be interesting to know that the problems on wedges are basically the problems of equilibrium on inclined planes. Thus these problems may be solved either by the equilibrium method or by applying Lami's theorem. Now consider a wedge *ABC*, which is used to lift the body *DEFG*.

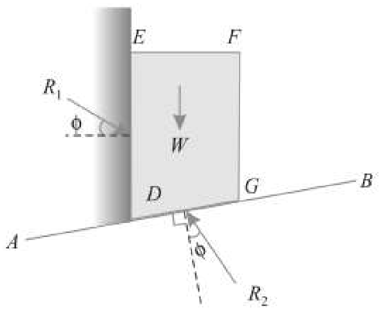
Let *W* = Weight fo the body *DEFG*,

P = Force required to lift the body, and

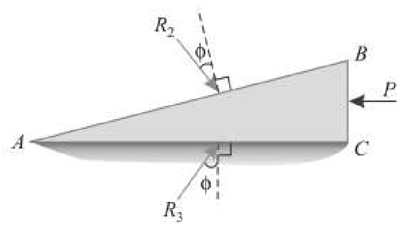
μ = Coefficient of friction on the planes *AB*, *AC* and *DE* such that

tan ϕ = μ.

A little consideration will show that when the force is sufficient to lift the body, the sliding will take place along three planes *AB*, *AC* and *DE* will also occur as shown in Fig.(*a*) and (*b*).



(*a*) Forces on the body *DEFG*



(*a*) Forces on the wedge *ABC*

The three reactions and the horizontal force (*P*) may now be found out by analytical method as discussed below:

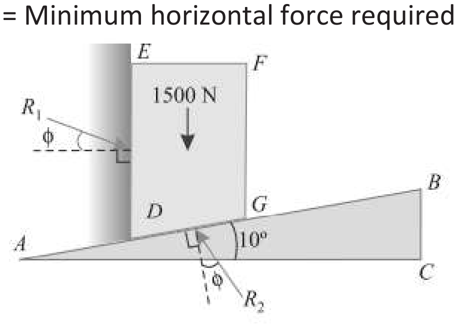
Analytical method:

1. First of all, consider the equilibrium of the body DEFG. And resolve the forces W, R1 and R2 horizontally as well as vertically.
2. Now consider the equilibrium of the wedge ABC. And resolve the forces P, R2 and R3 horizontally as well as vertically.

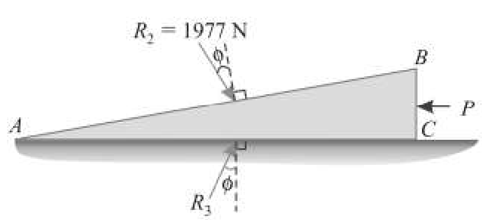
EXAMPLE: A block weighing 1500 N, overlying a 10° wedge on a horizontal floor and leaning against a vertical wall, is to be raised by applying a horizontal force to the wedge. Assuming the coefficient of friction between all the surface in contact to be 0.3, determine the minimum horizontal force required to raise the block.

SOLUTION: Given: Weight of the block (*W*) = 1500 N; Angle of the wedge (α) = 10° and coefficient of friction between all the four surfaces of contact (μ) = 0.3 = tan ϕ or ϕ = 16.7°

Let *P* = Minimum horizontal force required to raise the block.



(*a*) Block *DEFG*



(*b*) Wedge *ABC*

First of all, consider the equilibrium of the block. We know that it is in equilibrium under the action of the following forces as shown in Fig. (*a*).

1. Its own weight 1500 N acting downwards.
2. Reaction *R*1 on the face *DE*.
3. Reaction *R*2 on the face *DG* of the block.

Resolving the forces horizontally,

$$R_1 \cos(16.7°) = R_2 \sin(10 + 16.7°) = R_2 \sin 26.7°$$

$$R_1 \times 0.9578 = R_2 \times 0.4493$$

or $$R_2 = 2.132\, R_1$$

and now resolving the forces vertically,

$$R_1 \times \sin(16.7°) + 1500 = R_2 \cos(10° + 16.7°) = R_2 \cos 26.7°$$

$$R_1 \times 0.2874 + 1500 = R_2 \times 0.8934 = (2.132\, R_1)0.8934$$

$$= 1.905\, R_1 \qquad ...(R_2 = 2.132\, R_1)$$

$$R_1(1.905 - 0.2874) = 1500$$

$$\therefore \quad R_1 = \frac{1500}{1.6176} = 927.3 \text{ N}$$

and $$R_2 = 2.132\, R_1 = 2.132 \times 927.3 = 1977 \text{ N}$$

Now consider the equilibrium of the wedge. We know that it is in equilibrium under the action of the following forces as shown in Fig. 9.14 (*b*).

1. Reaction R_2 of the block on the wedge.
2. Force (P) acting horizontally, and
3. Reaction R_3 on the face *AC* of the wedge.

Resolving the forces vertically,

$$R_3 \cos 16.7° = R_2 \cos(10° + 16.7°) = R_2 \cos 26.7°$$

$$R_3 \times 0.9578 = R_2 \times 0.8934 = 1977 \times 0.8934 = 1766.2$$

$$\therefore \quad R_3 = \frac{1766.2}{0.9578} = 1844 \text{ N}$$

and now resolving the forces horizontally,

$$P = R_2 \sin(10° + 16.7°) + R_3 \sin 16.7° = 1977 \sin 26.7° + 1844 \sin 16.7° \text{ N}$$

$$= (1977 \times 0.4493) + (1844 \times 0.2874) = 1418.3 \text{ N} \qquad \textbf{Ans.}$$