1. FUNDAMENTALS OF ENGINEERING MECHANICS

ENGINEERING MECHANICS: The subject of Engineering Mechanics is that branch of Applied Science, which deals with the laws and principles of Mechanics, along with their applications to engineering problems.

The subject of Engineering Mechanics may be divided into the following two main groups: 1. Statics, and 2. Dynamics

STATICS: It is that branch of Engineering Mechanics, which deals with the forces and their effects, while acting upon the bodies at rest.

DYNAMICS: It is that branch of Engineering Mechanics, which deals with the forces and their effects, while acting upon the bodies in motion. The subject of Dynamics may be further sub-divided into the following two branches:
1. Kinetics, and 2. Kinematics

KINETICS: It is the branch of Dynamics, which deals with the bodies in motion due to the application of forces.

KINEMATICS: It is that branch of Dynamics, which deals with the bodies in motion, without any reference to the forces which are responsible for the motion.

RIGID BODY: A rigid body (also known as a rigid object) is a solid body in which deformation is zero or so small it can be neglected. The distance between any two given points on a rigid body remains constant in time regardless of external forces exerted on it. A rigid body is usually considered as a continuous distribution of mass.

FORCE: It is defined as an agent which produces or tends to produce, destroys or tends to destroy motion. *e.g.*, a horse applies force to pull a cart and to set it in motion. Force is also required to work on a bicycle pump. In this case, the force is supplied by the muscular power of our arms and shoulders.

SYSTEM OF FORCES: When two or more forces act on a body, they are called to form a system of forces. Following systems of forces are important from the subject point of view;
1. **Coplanar forces**: The forces, whose lines of action lie on the same plane, are known as coplanar forces.
2. **Collinear forces**: The forces, whose lines of action lie on the same line, are known as collinear forces
3. **Concurrent forces:** The forces, which meet at one point, are known as concurrent forces. The concurrent forces may or may not be collinear.
4. **Coplanar concurrent forces**: The forces, which meet at one point and their lines of action also lie on the same plane, are known as coplanar concurrent forces.
5. **Coplanar non-concurrent forces:** The forces, which do not meet at one point, but their lines of action lie on the same plane, are known as coplanar non-concurrent forces.
6. **Non-coplanar concurrent forces**: The forces, which meet at one point, but their lines of

action do not lie on the same plane, are known as non-coplanar concurrent forces.
7. **Non-coplanar non-concurrent forces**: The forces, which do not meet at one point and their lines of action do not lie on the same plane, are called non-coplanar non-concurrent forces.

CHARACTERISTIC OF A FORCE: In order to determine the effects of a force, acting on a body, we must know the following characteristics of a force:
1. Magnitude of the force (*i.e.*, 100 N, 50 N, 20 kN, 5 kN, etc.)
2. The direction of the line, along which the force acts (*i.e.*, along *OX*, *OY*, at 30° North of East etc.). It is also known as line of action of the force.
3. Nature of the force (*i.e.*, whether the force is push or pull). This is denoted by placing an arrow head on the line of action of the force.
4. The point at which (or through which) the force acts on the body

EFFECTS OF A FORCE: A force may produce the following effects in a body, on which it acts:
1. It may change the motion of a body. i.e. if a body is at rest, the force may set it in motion. And if the body is already in motion, the force may accelerate it.
2. It may retard the motion of a body.
3. It may retard the forces, already acting on a body, thus bringing it to rest or in equilibrium.
4. It may give rise to the internal stresses in the body, on which it acts.

PRINCIPLE OF TRANSMISSIBILITY: It states, "If a force acts at any point on a rigid body, it may also be considered to act at any other point on its line of action, provided this point is rigidly connected with the body."

PRINCIPLE OF SUPERPOSITION: This principle states that the combined effect of force system acting on a particle or a rigid body is the sum of effects of individual forces.

ACTION AND REACTION FORCE: Forces always act in pairs and always act in opposite directions. When you push on an object, the object pushes back with an equal force. Think of a pile of books on a table. The weight of the books exerts a downward force on the table. This is the action force. The table exerts an equal upward force on the books. This is the reaction force.

FREE BODY DIAGRAM: A free body diagram is a graphical illustration used to visualize the applied forces, moments, and resulting reactions on a body in a given condition. They depict a body or connected bodies with all the applied forces and moments, and reactions, which act on the body. The body may consist of multiple internal members (such as a truss), or be a compact body (such as a beam). A series of free bodies and other diagrams may be necessary to solve complex problems.

REOSLUTION OF A FORCE: The process of splitting up the given force into a number of components, without changing its effect on the body is called resolution of a force. A force is, generally, resolved along two mutually perpendicular directions.

COMPOSITION OF FORCES: The process of finding out the resultant force, of a number of given forces, is called composition of forces or compounding of forces.

RESULTANT FORCE: If a number of forces, P, Q, R ... etc. are acting simultaneously on a particle, then it is possible to find out a single force which could replace them i.e., which would produce the same effect as produced by all the given forces. This single force is called resultant force and the given forces R ...etc. are called component forces

METHODS FOR THE RESULTANT FORCE:
Though there are many methods for finding out the resultant force of a number of given forces, yet the following are important from the subject point of view :
1. Analytical method. 2. Method of resolution.

ANALYTICAL METHOD FOR RESULTANT FORCE:
The resultant force, of a given system of forces, may be found out analytically by the following methods :
1. Parallelogram law of forces. 2. Method of resolution.

PARALLELOGRAM LAW OF FORCES:
It states, "If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram ; their resultant may be represented in magnitude and direction by the diagonal of the parallelogram, which passes through their point of intersection."
Mathematically, resultant force,

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

and
$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2\cos\theta}$$

where F_1 and F_2 = Forces whose resultant is required to be found out,
θ = Angle between the forces F_1 and F_2, and
α = Angle which the resultant force makes with one of the forces (say F_1).

EXAMPLE: Two forces of 100 N and 150 N are acting simultaneously at a point. What is the resultant of these two forces, if the angle between them is 45°?

Solution. Given : First force (F_1) = 100 N; Second force (F_2) = 150 N and angle between F_1 and F_2 (θ) = 45°.
We know that the resultant force,

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$
$$= \sqrt{(100)^2 + (150)^2 + 2\times100\times150\cos 45°}\ \text{N}$$
$$= \sqrt{10\,000 + 22\,500 + (30\,000\times0.707)}\ \text{N}$$
$$= 232\ \text{N} \quad \textbf{Ans.}$$

EXAMPLE: Find the magnitude of the two forces, such that if they act at right angles, their resultant is 10 N . But if they Act at 60°, their resultant is 13 N .

Solution. Given : Two forces = F_1 and F_2.

First of all, consider the two forces acting at right angles. We know that when the angle between the two given forces is 90°, then the resultant force (R)

$$\sqrt{10} = \sqrt{F_1^2 + F_2^2}$$

or $$10 = F_1^2 + F_2^2 \quad \text{...(Squaring both sides)}$$

Similarly, when the angle between the two forces is 60°, then the resultant force (R)

$$\sqrt{13} = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos 60°}$$

$$\therefore \quad 13 = F_1^2 + F_2^2 + 2F_1 F_2 \times 0.5 \quad \text{...(Squaring both sides)}$$

or $$F_1 F_2 = 13 - 10 = 3 \quad \text{...(Substituting } F_1^2 + F_2^2 = 10)$$

We know that $(F_1 + F_{2]})^2 = F_1^2 + F_2^2 + 2F_1F_2 = 10 + 6 = 16$

$$\therefore \quad F_1 + F_2 = \sqrt{16} = 4 \quad \text{...}(i)$$

Similarly $(F_1 - F_2)^2 = F_1^2 + F_2^2 - 2F_1F_2 = 10 - 6 = 4$

$$\therefore \quad F_1 - F_2 = \sqrt{4} = 2 \quad \text{...}(ii)$$

Solving equations (i) and (ii),

$$F_1 = 3 \text{ N} \quad \text{and} \quad F_2 = 1 \text{ N} \quad \textbf{Ans.}$$

RESOLUTION OF A FORCE: The process of splitting up the given force into a number of components, without changing its effect on the body is called resolution of a force. A force is, generally, resolved along two mutually perpendicular directions. In fact, the resolution of a force is the reverse action of the addition of the component vectors.

PRINCIPLE OF RESOLUTION: It states, "The algebraic sum of the resolved parts of a no. of forces, in a given direction, is equal to the resolved part of their resultant in the same direction."

Note: In general, the forces are resolved in the vertical and horizontal directions.

METHOD OF RESOLUTION:

- Resolve all the forces horizontally and find the algebraic sum of all the horizontal components .
- Resolve all the forces vertically and find the algebraic sum of all the vertical components
- The resultant *R* of the given forces will be given by the equation;

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

- The resultant force will be inclined at an angle , with the horizontal, such that

$$\tan \theta = \frac{\Sigma V}{\Sigma H}$$

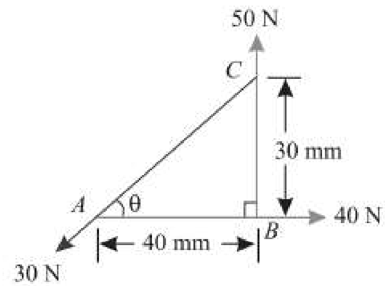
EXAMPLE: A triangle ABC has its side AB = 40 mm along positive x-axis and side BC = 30 mm along positive y-axis. Three forces of 40 N, 50 N and 30 N act along the sides AB, BC and CA respectively. Determine magnitude of the resultant of such a system of forces.

Solution. The system of given forces is shown in Fig. 2.3.

From the geometry of the figure, we find that the triangle *ABC* is a right angled triangle, in which the *side $AC = 50$ mm. Therefore

$$\sin\theta = \frac{30}{50} = 0.6$$

and $$\cos\theta = \frac{40}{50} = 0.8$$



Resolving all the forces horizontally (*i.e.*, along *AB*),

$$\Sigma H = 40 - 30\cos\theta$$
$$= 40 - (30 \times 0.8) = 16 \text{ N}$$

and now resolving all the forces vertically (*i.e.*, along *BC*)

$$\Sigma V = 50 - 30\sin\theta$$
$$= 50 - (30 \times 0.6) = 32 \text{ N}$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(16)^2 + (32)^2} = 35.8 \text{ N} \quad \textbf{Ans.}$$

LAWS FOR THE RESULTANT FORCE:

The resultant force, of a given system of forces, may also be found out by the following laws **1.** Triangle law of forces. **2.** Polygon law of forces.

TRIANGLE LAW OF FORCES:

It states, "If two forces acting simultaneously on a particle, be represented in magnitude and direction by the two sides of a triangle, taken in order ; their resultant may be represented in magnitude and direction by the third side of the triangle, taken in opposite order."

POLYGON LAW OF FORCES:

It is an extension of Triangle Law of Forces for more than two forces, which states, "If a number of forces acting simultaneously on a particle, be represented in magnitude and direction, by the sides of a polygon taken in order ; then the resultant of all these forces may be represented, in magnitude and direction, by the closing side of the polygon, taken in opposite order."

GRAPHICAL (VECTOR) METHOD FOR THE RESULTANT FORCE:

It is another name for finding out the magnitude and direction of the resultant force by the polygon law of forces. It is done as discussed below:

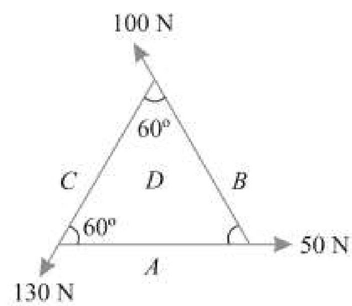
- **Construction of space diagram (position diagram)**: It means the construction of a diagram showing the various forces (or loads) along with their magnitude and lines of action.

- **Use of Bow's notations:** All the forces in the space diagram are named by using the Bow's notations. It is a convenient method in which every force (or load) is named by two capital letters, placed on its either side in the space diagram.
- **Construction of vector diagram (force diagram):** It means the construction of a diagram starting from a convenient point and then go on adding all the forces vectorially one by one (keeping in view the directions of the forces) to some suitable scale. Now the closing side of the polygon, taken in opposite order, will give the magnitude of the resultant force (to the scale) and its direction.

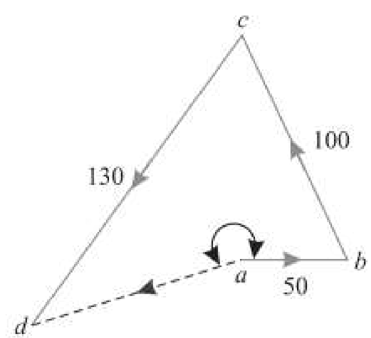
EXAMPLE: A particle is acted upon by three forces equal to 50 N, 100 N and 130 N, along the three sides of an equilateral triangle, taken in order. Find graphically the Magnitude and direction of the resultant force.

Solution. The system of given forces is shown in Fig. 2.8 (*a*)

First of all, name the forces according to Bow's notations as shown in Fig. 2.8 (*a*). The 50 N force is named as *AD*, 100 N force as *BD* and 130 N force as *CD*.



(*a*) Space diagram



(*b*) Vector diagram

Now draw the vector diagram for the given system of forces as shown in Fig. 2.8 (*b*) and as discussed below :

1. Select some suitable point *a* and draw *ab* equal to 50 N to some suitable scale and parallel to the 50 N force of the space diagram.
2. Through *b*, draw *bc* equal to 100 N to the scale and parallel to the 100 N force of the space diagram.
3. Similarly through *c*, draw *cd* equal to 130 N to the scale and parallel to the 130 N force of the space diagram.
4. Join *ad*, which gives the magnitude as well as direction of the resultant force.
5. By measurement, we find the magnitude of the resultant force is equal to 70 N and acting at an angle of 200° with *ab*. **Ans.**

MOMENT OF A FORCE: It is the turning effect produced by a force, on the body, on which it acts. The moment of a force is equal to the product of the force and the perpendicular distance of the point, about which the moment is required and the line of action of the force.

Mathematically, moment,

$M = P \times l$

where P = Force acting on the body, and

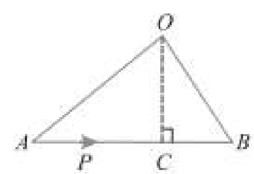
l = Perpendicular distance between the point, about which the moment is required and the line of action of the force.

GRAPHICAL REPRESENTATION OF A MOMENT: Consider a force *P* represented, in magnitude and direction, by the line *AB*. Let *O* be a point, about which the moment of this force is required to be found out, as shown in Fig. From *O*, draw *OC* perpendicular to *AB*. Join *OA* and *OB*.

Now moment of the force *P* about *O*

$$= P \times OC = AB \times OC$$

But *AB* × *OC* is equal to twice the area of triangle *ABO*. Thus the moment of a force, about any point, is equal to twice the area of the triangle, whose base is the line to some scale representing the force and whose vertex is the point about which the moment is taken.



UNITS OF MOMENT:

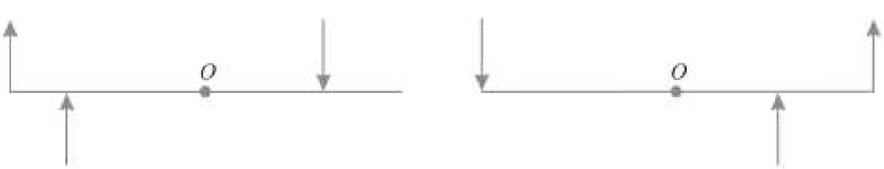
Since the moment of a force is the product of force and distance, therefore the units of the moment will depend upon the units of force and distance. Thus, if the force is in Newton and the distance is in meters, then the units of moment will be Newton-meter (briefly written as N-m). Similarly, the units of moment may be kN-m (*i.e.* kN × m), N-mm (*i.e.* N × mm) etc.

TYPES OF MOMENTS:

Broadly speaking, the moments are of the following two types:

1. Clockwise moments. 2. Anticlockwise moments.

CLOCKWISE MOMENT:



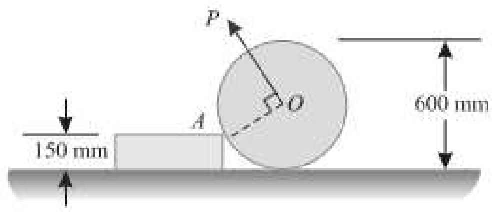
(*a*) Clockwise moments (*b*) Anticlockwise moments

It is the moment of a force, whose effect is to turn or rotate the body, about the point in the same direction in which hands of a clock move as shown in Fig.

ANTICLOCKWISE MOMENT:
It is the moment of a force, whose effect is to turn or rotate the body, about the point in the opposite direction in which the hands of a clock move as shown in Fig.(*b*). **Note.** The general convention is to take clockwise moment as positive and anticlockwise moment as negative.

VARIGNON'S PRINCIPLE OR LAW OF MOMENTS:
It states, "If a number of coplanar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point."

EXAMPLE: A uniform wheel of 600 mm diameter, weighing 5 kN rests against a rigid rectangular block of 150 mm height as shown in Fig.



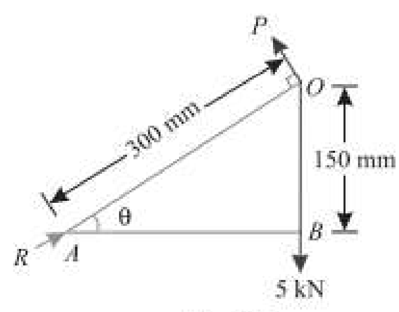
Find the least pull, through the centre of the wheel, required just to turn the wheel over the corner A of the block. Also find the reaction on the block. Take all the surfaces to be smooth.

Solution. Given : Diameter of wheel = 600 mm; Weight of wheel = 5 kN and height of the block = 150 mm.

Least pull required just to turn the wheel over the corner.

Let P = Least pull required just to turn the wheel in kN.

A little consideration will show that for the least pull, it must be applied normal to AO. The system of forces is shown in Fig. 3.9. From the geometry of the figure, we find that



$$\sin\theta = \frac{150}{300} = 0.5 \quad \text{or} \quad \theta = 30°$$

and
$$AB = \sqrt{(300)^2 - (150)^2} = 260 \text{ mm}$$

Now taking moments about A and equating the same,

$$P \times 300 = 5 \times 260 = 1300$$

$$\therefore \quad P = \frac{1300}{300} = 4.33 \text{ kN} \quad \textbf{Ans.}$$

Reaction on the block

Let R = Reaction on the block in kN.

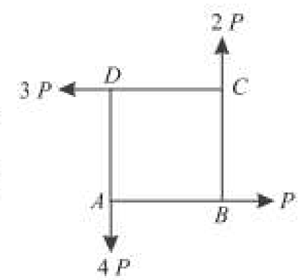
Resolving the forces horizontally and equating the same,

$$R\cos 30° = P\sin 30°$$

$$\therefore \quad R = \frac{P\sin 30°}{\cos 30°} = \frac{4.33 \times 0.5}{0.866} = 2.5 \text{ kN} \quad \textbf{Ans.}$$

EXAMPLE: Four forces equal to P, 2P, 3P and 4P are respectively acting along the four sides of square ABCD taken in order. Find the magnitude, direction and position of the resultant force.

Solution. The system of given forces is shown in Fig. 3.12.



Magnitude of the resultant force

Resolving all the forces horizontally,

$$\Sigma H = P - 3P = -2P \quad ...(i)$$

and now resolving all forces vertically,

$$\Sigma V = 2P - 4P = -2P \quad ...(ii)$$

We know that magnitude of the resultant forces,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(-2P)^2 + (-2P)^2}$$

$$= 2\sqrt{2}P \quad \textbf{Ans.}$$

Direction of the resultant force

Let θ = Angle, which the resultant makes with the horizontal.

$$\therefore \quad \tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{-2P}{-2P} = 1 \quad \text{or} \quad \theta = 45^\circ$$

Since ΣH as well as ΣV are –ve, therefore resultant lies between 180°and 270°. Thus actual angle of the resultant force = 180° + 45° = 225° **Ans.**

Position of the resultant force

Let x = Perpendicular distance between A and the line of action of the resultant force.

Now taking moments of the resultant force about A and equating the same,

$$2\sqrt{2}P \times x = (2P \times a) + (3P \times a) = 5P \times a$$

$$\therefore \quad x = \frac{5a}{2\sqrt{2}} \quad \textbf{Ans.}$$

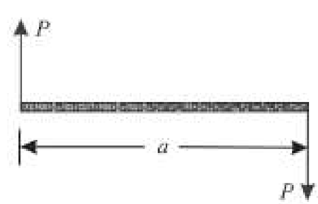
Note. The moment of the forces P and $4P$ about the point A will be zero, as they pass through it.

COUPLE: A pair of two equal and unlike parallel forces (*i.e.*forces equal in magnitude, with lines of action parallel to each other and acting in opposite directions) is known as a couple.

As a matter of fact, a couple is unable to produce any translatory motion (*i.e.*, motion in a straight line). But it produces a motion of rotation in the body, on which it acts. The simplest example of a couple is the forces applied to the key of a lock, while locking or unlocking it.

ARM OF A COUPLE:

The perpendicular distance (*a*), between the lines of action of the two equal and opposite parallel forces, is known *as arm of the couple* as shown in Fig.

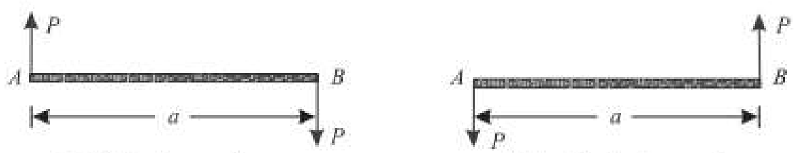


MOMENT OF A COUPLE: The moment of a couple is the product of the force (*i.e.*, one of the forces of the two equal and opposite parallel forces) and the arm of the couple.
Mathematically:
Moment of a couple = $P \times a$

where P = Magnitude of the force, and
a = Arm of the couple.

CLASSIFICATION OF COUPLES:
The couples may be, broadly, classified into the following two categories, depending upon their direction, in which the couple tends to rotate the body, on which it acts:

1. Clockwise couple, and 2. Anticlockwise couple



(*a*) Clockwise couple (*b*) Anticlockwise couple

CLOCKWISE COUPLE:
A couple, whose tendency is to rotate the body, on which it acts, in a clockwise direction, is known as a clockwise couple as shown in Fig. (*a*). Such a couple is also called positive couple.

ANTICLOCKWISE COUPLE:
A couple, whose tendency is to rotate the body, on which it acts, in an anticlockwise direction, is known as an anticlockwise couple as shown in Fig. (*b*). Such a couple is also called a negative couple.

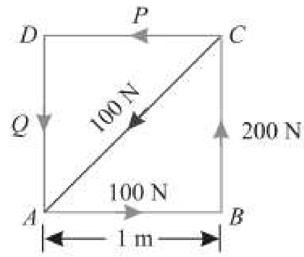
CHARACTERISTICS OF A COUPLE:
A couple (whether clockwise or anticlockwise) has the following characteristics:

1. The algebraic sum of the forces, constituting the couple, is zero.
2. The algebraic sum of the moments of the forces, constituting the couple, about any point is the same, and equal to the moment of the couple itself.
3. A couple cannot be balanced by a single force. But it can be balanced only by a couple of opposite sense.
4. Any no. of co-planer couples can be reduced to a single couple, whose magnitude will be equal to the algebraic sum of the moments of all the couples.

EXAMPLE: A square ABCD has forces acting along its sides as shown in Fig. 4.13. Find the values of P and Q, if the system reduces to a couple. Also find magnitude of the couple, if the side of the square is 1 m.

Solution. Given : Length of square = 1 m

Values of P and Q



We know that if the system reduces to a couple, the resultant force in horizontal and vertical directions must be zero. Resolving the forces horizontally,

$$100 - 100 \cos 45° - P = 0$$

$$\therefore \quad P = 100 - 100 \cos 45° \text{ N}$$

$$= 100 - (100 \times 0.707) = 29.3 \text{ N} \quad \textbf{Ans.}$$

Now resolving the forces vertically,

$$200 - 100 \sin 45° - Q = 0$$

$$\therefore \quad Q = 200 - (100 \times 0.707) = 129.3 \text{ N} \quad \textbf{Ans.}$$

Magnitude of the couple

We know that moment of the couple is equal to the algebraic sum of the moments about any point. Therefore moment of the couple (taking moments about *A*)

$$= (-200 \times 1) + (-P \times 1) = -200 - (29.3 \times 1) \text{ N-m}$$

$$= -229.3 \text{ N-m} \quad \textbf{Ans.}$$

Since the value of moment is negative, therefore the couple is anticlockwise.