

# Centroid and centre of gravity.

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Centre of gravity - It is the point where entire weight of the system or body is assumed to be concentrated.

Centre of Mass :- It is the point where entire mass of system or body is assumed to be concentrated.

Centroid! These are geometrical centres, defined for 3-D, 2D and 1D objects, where entire volume, area and length are assumed to be concentrated. called

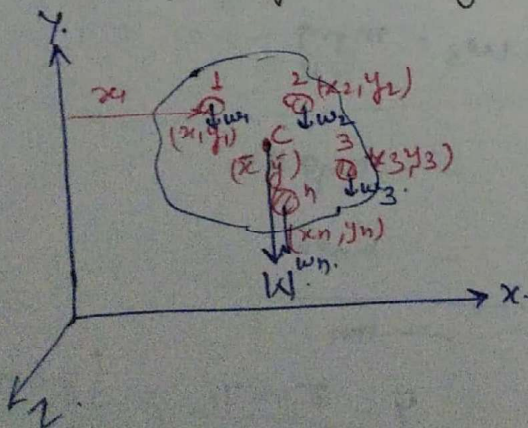
COV [centroid of volume] - for space

COA [ " " " Area] - for Area

COL [ " " " Length] - for - Line.

\* If the value of gravity is uniform for each part and the solid is of uniform density throughout, then the centroid, centre of gravity are centre of mass are coincident.

Centre of gravity of any body or system! -



Weight of body  $W$

Mass of body  $M$

Area of body  $A$

Volume " "  $V$

density -  $\rho$ .

$$W = w_1 + w_2 + w_3 + \dots + w_n$$

$$M = m_1 + m_2 + m_3 + \dots + m_n$$

Varignon's Theorem:

$$R \times d = F_1 d_1 + F_2 d_2 + \dots + F_n d_n$$

Similarly, Varignon's Theorem about y axis. for moment of weight

$$W \times \bar{x} = w_1 x_1 + w_2 x_2 + w_3 x_3 + \dots + w_n x_n$$

$$\bar{x} = \frac{w_1 x_1 + w_2 x_2 + \dots + w_n x_n}{W}$$

$$= \frac{w_1 x_1 + w_2 x_2 + \dots + w_n x_n}{w_1 + w_2 + \dots + w_n}$$

$$\bar{x} = \frac{\sum w_i x_i}{\sum w_i} \quad \& \quad \bar{x} = \frac{\int x \cdot dw}{\int dw}$$

Similarly  $\rightarrow \bar{y} = \frac{\sum w_i y_i}{\sum w_i} \quad \& \quad \bar{y} = \frac{\int y \cdot dw}{\int dw}$

If the gravitational acceleration ( $g$ ) will be constant or uniform throughout the body or system. then the C.O.M and C.O.G will be the same.

$$W = M \cdot g$$

$$w_1 = m_1 g, \quad w_2 = m_2 g, \quad w_3 = m_3 g, \dots$$

$$M \times g \times \bar{x} = m_1 g x_1 + m_2 g x_2 + \dots + m_n g x_n$$

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2 + \dots + m_n x_n}{m_1 + m_2 + \dots + m_n}$$

$$\bar{x} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\int x \cdot dm}{\int dm}, \quad \bar{y} = \frac{\sum m_i y_i}{\sum m_i} = \frac{\int y \cdot dm}{\int dm}$$

all sections are of same material ( $\rho$  density).

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$$m_1 = \rho V_1, \quad m_2 = \rho V_2, \quad m_3 = \rho V_3.$$

$$\text{total Volume} = V_1 + V_2 + V_3 + \dots + V_n.$$

$$\bar{x} = \frac{\sum \rho V \cdot x}{\sum \rho V} = \frac{\sum Vx}{\sum V}$$
$$\bar{y} = \frac{\sum \rho V \cdot y}{\sum \rho V} = \frac{\sum Vy}{\sum V}$$

~~by~~ Systems/bodies are of same cross section, but different lengths.

$$V_1 = a l_1, \quad V_2 = a l_2, \quad \dots$$

$$V = a l_1 + a l_2 + \dots$$

$$\bar{x} = \frac{\sum a l x}{\sum a l} = \frac{\sum l x}{\sum l}$$

$$\bar{y} = \frac{\sum a l \cdot y}{\sum a l} = \frac{\sum l y}{\sum l}$$