e-Class Notes

Measure of Central Tendency and Arithmetic Mean

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Measure of Central Tendency

Measures of central tendency are sometimes called measures of central location. A measure of central tendency is a single value that attempts to describe a set of data by identifying the central position within that set of data. Plainly speaking, an average of statistical series is the value of the variable which is representative of the entire distribution. The following are the five measure of Central tendency are

(i) Arithmetic Mean or Mean

(ii) Median

(iii) Mode

(iv) Geometric mean

and (V) Harmonic mean

Requirements of good measures of Central tendency:

The following are the characteristics to be satisfied by an ideal measure of Central tendency

(i) It should be rigidly defined

(ii) It should be based on all the observations

(iii) It should be readily the comprehensible and easy to calculate

(iv) It should be suitable for further mathematical treatment

(v) It should be affected as little as possible by fluctuation of sampling

(vi) It should not be affected much by extreme values

Arithmetic Mean:

The arithmetic is the simplest and most widely used measure of a mean, or average it simply involves taking the sum of a group of numbers, then diving that sum by the count of the numbers used in series.

Method I: Direct Method

The Arithmetic mean $\bar{x}$ of n observations ($x_1, x_2, x_3, \ldots\ldots\ldots, x_n$) is given by

$$\text{Arithmetic Mean } \bar{x} = \frac{x_1 + x_2 + x_3 + \ldots\ldots\ldots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\bar{x} = \frac{Sum\ of\ \ Observations}{Total\ \ No.of\ Observations}$$

Method II: Shortcut Method

$$\text{Arithmetic Mean } \ \bar{x} = A + \frac{1}{n}\sum_{i=1}^{n} d_i$$

where $d_i = x_i - A$

A =Assumed mean

In case of frequency distribution,

Method –I (Direct Method)

The Arithmetic mean is obtained by

$$\bar{x} = \frac{f_1\, x_1 + f_2\, x_2 + f_3\, x_3 + \ldots\ldots\ldots + f_n\, x_n}{f_1 + f_2 + f_3 + \ldots\ldots\ldots\ldots\ldots + f_n} = \frac{1}{N}\sum_{i=1}^{n} f_i\, x_i = \frac{1}{N}\sum f_i\, x_i \qquad \left[\sum_{i=1}^{n} f_i = N\right]$$

where f_i is the frequency of the variable x_i.

Method –II (Shortcut Method)

Arithmetic mean $\bar{x} = A + \dfrac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i}$

$d_i = x_i - A$

A =Assumed mean

Method –III (Step-Deviation Method)

Arithmetic Mean $\bar{x} = A + h \times \dfrac{\sum_{i=1}^{n} f_i d_i'}{\sum_{i=1}^{n} f_i} = A + h \dfrac{\sum f_i d_i'}{\sum f_i}$

Where $d_i' =$ Step Deviation $= d_i' = \dfrac{x_i - A}{h}$

A =Assumed mean

h = interval

In case of grouped or continuous frequency distribution, x is taken as the mid value of the corresponding class.

Remark. The symbol $\sum$ is used in Mathematics to denote the sum of values.

Illustration1. Find the arithmetic mean of the following marks obtained by 10 students in statistics.

51, 41, 70, 75, 43, 40, 35, 65, 50, 60

Sol. Shortcut Method:

Let Assumed mean A = 50

x_i	$d_i = x_i - A$
51	1
41	-9
70	20
75	25
43	-7
40	-10
35	-15
65	15
50	0
60	10
	$\sum d_i = 30$

$$\bar{x} = A + \frac{\sum d_i}{n}$$

$$\bar{x} = 50 + \frac{30}{10} = 53\, marks$$

Illustration2. Following table gives wages paid to 125 workers in a factory calculate the arithmetic mean of the wages.

Wages	Number of workers
200	5
210	15
220	32
230	42
240	15
250	12
260	4
Total	125

Sol. Let Assumed Mean A = 230

Wages x_i	Number of workers f_i	$d_i = x_i - A$	$f_i\, d_i$
200	5	-30	-150
210	15	-20	-300
220	32	-10	-320
230	42	0	0
240	15	10	150
250	12	20	240
260	4	30	120
	$\sum f_i$ =125		$\sum f_i\, d_i = -260$

Arithmetic Mean $\bar{x} = A + \frac{\sum f_i d_i}{\sum f_i} = 230 + \frac{(-260)}{125}$

$=227.9$

Illustration3. Find the arithmetic mean for the following distribution:

Class Interval	Frequency
0-10	7
10-20	8
20-30	20
30-40	10
40-50	5

Sol. Arithmetic Mean $= A + h \times \frac{\sum f_i d_i'}{N}$

where f_i =frequency

x_i =Mid value of class interval

Assumed mean A = 25

$N = \sum f_i$

h = interval

Class Interval	Frequency (f_i)	Mid Value (x_i)	$d_i = x_i - A$	$d_i' = \frac{x_i - A}{h}$	$f_i\, d_i'$
0-10	7	5	- 20	-2	-14
10-20	8	15	-10	-1	-8
20-30	20	25	0	0	0
30-40	10	35	10	1	10
40-50	5	45	20	2	10
	$\sum f_i = 50$				$\sum f_i d_i' = -2$

Arithmetic Mean $= A + h \times \frac{\sum f_i\, d_i'}{N}$

$$= 25 + 10 \times \frac{-2}{50}$$

$$= 25 - 0.4$$

Arithmetic Mean $= 24.6$

Combined mean:

If the numbers of observations of two or more related groups are known, we can calculate the combined arithmetic mean of these groups.

Formula for two related groups

$$\bar{X}_{12} = \frac{\bar{x}_1 N_1 + \bar{x}_2 N_2}{N_1 + N_2}$$

N_1 = Number of observations in first group

N_2 = Number of observations in second group

$\bar{x}_1$ = Mean of the first group

$\bar{x}_2$ = Mean of second group

$\bar{X}_{12}$ = Combined mean of two group

Similarly, Formula for three related groups

$$\bar{X}_{123} = \frac{\bar{x}_1 N_1 + \bar{x}_2 N_2 + \bar{x}_3 N_3}{N_1 + N_2 + N_3}$$

Illustration4. The mean marks of 60 students in a section A is 40, and mean marks of 40 students in section B is 45. Find combined mean of 100 students in both the sections.

Sol. Let

N_1 = Number of students in section A

N_2 = Number of student in section B

$\bar{x}_1$ = Mean marks of students in section A

$\bar{x}_2$ = Mean marks of students in section B

$\bar{X}_{12} =$ Combined mean of students in both the sections

Formula

Combined mean $\bar{X}_{12} = \dfrac{\bar{x}_1 N_1 + \bar{x}_2 N_2}{N_1 + N_2}$

$N_1 = 60$

$N_2 = 40$

$\bar{x}_1 = 40$

$\bar{x}_2 = 45$

$$\bar{X}_{12} = \frac{60 \times 40 + 40 \times 45}{60 + 40}$$

$$\bar{X}_{12} = \frac{40(60 + 45)}{100}$$

$$\bar{X}_{12} = \frac{40 \times 105}{100} = \frac{4200}{100}$$

$$\bar{X}_{12} = 42$$

Illustration5. The mean annual salary of employees was 3000, mean annual salary paid to male and female employees 3200 and 2200 respectively determine the percentage of male and female employees.

Sol. Let N_1 = Number of male employees

N_2 = Number of female employees

$\bar{x}_1 =$ Mean annual salary of male employees

$\bar{x}_2 =$ Mean annual salary of female employees

$\bar{X}_{12} =$ Mean annual salary of employees

<u>Formula</u>

$$\bar{X}_{12} = \frac{\bar{x}_1 N_1 + \bar{x}_2 N_2}{N_1 + N_2}$$

$\bar{x}_1 = 3200$

$\bar{x}_2 = 2200$

$\bar{X}_{12} = 3000$

$$3000 = \frac{3200 N_1 + 2200 N_2}{N_1 + N_2}$$

$$30(N_1 + N_2) = 32 N_1 + 22 N_2$$

$$30 N_1 + 30 N_2 = 32 N_1 + 22 N_2$$

$$8 N_2 = 2 N_1$$

$$\frac{N_1}{N_2} = \frac{4}{1}$$

Hence, the percentage of male employees $= \frac{4}{5} \times 100 = 80\%$,

The percentage of female employees $= \frac{1}{5} \times 100 = 20\%$

Correcting the Arithmetic Mean:

For correcting the incorrect value of mean, first we find the corrected $\sum x_i$. For this we subtract the wrong items from the incorrect $\sum x_i$ and add it to the correct items. Finally, on dividing the corrected $\sum x_i$ by the number of the observations, we get the corrected mean

Formula:

Correct Sum $\sum x_i$ = Incorrect Sum $\sum x_i -$ (Sum of Incorrect values) + (Sum of correct values)

Correct Mean = $\dfrac{Correct\ Sum}{Total\ Number\ of\ Observations}$

Illustration6. Average marks of 80 students were found to be 40. Later, it was discovered that a score of 64 was misread as 94. Find the corrected mean of the 80 students.

Sol. Incorrect mean of 80 students =40

i.e. $\dfrac{\sum x_i}{80} = 40$

Incorrect sum $\sum x_i = 80 \times 40 = 3200$

Incorrect total of the marks of 80 students $= 80 \times 40 = 3200$ marks

Formula

Corrected $\sum x_i$ = Incorrect $\sum x_i -$ (Sum of Incorrect values) + (Sum of correct values)

Therefore, the corrected total of the marks of 80 students=3200 - 94+64 = 3170

Corrected Mean = Corrected $\dfrac{\sum x_i}{n}$

The corrected average of 80 students $= \dfrac{3170}{80} = 39.625\,marks$

Exercises

1. Find the arithmetic mean of the following frequency distribution:

x	1	2	3	4	5	6	7
f	5	9	12	17	12	10	6

2. Calculate arithmetic mean from following distribution

Class Interval	Frequency (f_i)
10-19	5
20-29	12
30-39	15
40-49	20
50-59	18
60-69	6

3. The mean height of 25 male workers in a factory is 60 inches, and that of 35 female workers in a same factory is 58 inches, find the average height of all the workers in factory.

4. The mean of 100 items was 50. Later on it was discovered that two items were misread as 92 and 8 instead of 192 and 88 find the corrected mean

5. Mean of 100 items is found to be 30. If at the time of calculation, two items are wrongly taken as 32 and 12 instead of 23 and 11, find the correct mean.

References

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