

## **e-Class notes**

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Subject Name: Numerical Methods  
Subject Code: BCA 504(N)  
Topic: Lagrange's Interpolation Formula

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By

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## Lagrange's Interpolation formula for Unequal Intervals

If  $y = f(x)$  be a function with which takes  $(n+1)$  values of  $f(x_0), f(x_1), f(x_2) \dots f(x_n)$  corresponding to  $x = x_0, x_1, \dots, x_n$ , which are not necessarily equally-spaced, then  $f(x)$  can be represented as a polynomial of degree  $n$  in  $x$ .

$$f(x) = \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} f(x_0) + \frac{(x-x_0)(x-x_2)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)} f(x_1) + \frac{(x-x_0)(x-x_1)(x-x_3)\dots(x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\dots(x_2-x_n)} f(x_2) + \dots + \frac{(x-x_0)(x-x_1)\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\dots(x_n-x_{n-1})} f(x_n)$$

This formula is known as Lagrange's Interpolation formula.

Q1) Use Lagrange's formula to find the form of function  $f(x)$  given that

$x$	:	0	2	3	6
$f(x)$	:	648	704	729	792

Solu.  $\Rightarrow$  Here

$$x_0 = 0, x_1 = 2, x_2 = 3, x_3 = 6$$

$$f(x_0) = y_0 = 648, y_1 = 704 = f(x_1), f(x_2) = 729, f(x_3) = 792$$

Using Lagrange's formula, we have

$$\begin{aligned} f(x) &= \frac{(x-2)(x-3)(x-6)}{(0-2)(0-3)(0-6)} 648 + \frac{(x-0)(x-3)(x-6)}{(2-0)(2-3)(2-6)} 704 \\ &+ \frac{(x-0)(x-2)(x-6)}{(3-0)(3-2)(3-6)} 729 + \frac{(x-0)(x-2)(x-3)}{(6-0)(6-2)(6-3)} 792 \\ &= \frac{(x^3 - 11x^2 + 36x - 36)(-18)}{(88)} + \frac{(x^3 - 9x^2 + 18x)(-81)}{(81)} + \frac{(x^3 - 8x^2 + 12x)(-81)}{(81)} + \frac{(x^3 - 5x^2 + 6x)(11)}{(11)} \\ &= -x^2 + 30x + 648 \end{aligned}$$

Q2 → Using Lagrange's interpolation formula, find the value of  $f(10)$

$x$	5	6	9	11
$f(x)$	12	13	14	16

Solu →

Applying Lagrange's formula for  $x_0=5, x_1=6, x_2=9, x_3=11$  and  $x=10$  and  $f(x_0)=12, f(x_1)=13, f(x_2)=14, f(x_3)=16$ , we get

$$f(10) = \frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)} \times 12 + \frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)} \times 13$$

$$+ \frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)} \times 14 + \frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)} \times 16$$

$$= \frac{4 \times 1 \times -1}{-1 \times -4 \times -6} \times 12 + \frac{5 \times (1) \times (-1)}{(1) \times (-3) \times (-5)} \times 13 + \frac{5 \times 4 \times -1}{4 \times 3 \times -2} \times 14 + \frac{5 \times 4 \times 1}{6 \times 5 \times 2} \times 16$$

$$= 2 - 4.33 + 11.67 + 5.33$$

$$= 14.67$$

Q1 Find the form of function

$x$	0	1	2	5
$f(x)$	2	3	12	147

Q2: Use Lagrange's formula to find the value of  $\log_{10} 656$  by the following table.

$x$	654	658	659	661
$\log_e x$	2.8156	2.8182	2.8189	2.8202

## Reference Books

1. H.K. Dass – Advanced Engineering Mathematics – S. Chand & Co., 9th Revised Ed.
2. Gupta and S. C. Boss – Introduction to Numerical Analysis – Academic Press Kolkata