e-Class notes

Subject Name: Numerical Methods
Subject Code: BCA 504(N)
Topic: Newton's Divided Difference Formula

By

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Newton's Divided Difference formula →

If $f(x_0), f(x_1), f(x_2) - - - f(x_n)$ be the values of an nth degree Polynomial $f(x)$ corresponding to the arguments $x_0, x_1, - - - x_n$, which are not necessarily equally-spaced, then

$$f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2)$$
$$+ - - - - + (x-x_0)(x-x_1) - - - (x-x_{n-1}) f(x_0, x_1, - - - - x_n) \}$$

Ex-1 → By Newton's method, find $f(6)$, where

x	3	7	9	10
$f(x)$	168	120	72	63

Soln → The divided difference table

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
3	168	$\frac{120-168}{7-3} = -12$	$\frac{-24+12}{9-3} = -2$	$\frac{5+2}{10-3} = 1$
7	120	$\frac{72-120}{9-7} = -24$	$\frac{-9+24}{10-7} = 5$	
9	72	$\frac{63-72}{10-9} = -9$		
10	63			

Now using Newton's divided difference formula

$$f(x) = f(x_0) + (x-x_0) \Delta f(x_0) + (x-x_0)(x-x_1) \Delta^2 f(x_0) +$$
$$(x-x_0)(x-x_1)(x-x_2) \Delta^3 f(x_0)$$

$$= 168 + (x-3)(-12) + (x-3)(x-7)(-2) + (x-3)(x-7)(x-9)(1)$$

If $x = 6$

$$f(6) = 168 + (6-3)(-12) + 3 \times (6-7)(-2) + 3 \times (-1)(6-9)(1)$$

$$= 147$$

Q. Find $f(x)$ such that

x	0	1	4	6
$f(x)$	1	2	17	37

Solu. The divided difference table for the given data is as follows

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$
0	1		
1	2	$\frac{2-1}{1-0}=1$	$\frac{5-1}{4-0}=1$
4	17	$\frac{17-2}{4-1}=5$	$\frac{10-5}{6-1}=1$
6	37	$\frac{37-17}{6-4}=10$	

Now using Newton's divided difference formula, we have

$$f(x) = f(x_0) + (x-x_0)\Delta f(x_0) + (x-x_0)(x-x_1)\Delta^2 f(x)$$

$$= 1 + (x-0)\times 1 + (x-0)(x-1)\times 1 =$$
$$= 1 + x + (x^2 - x)$$
$$f(x) = x^2 + 1$$

Related Questions

Q1 → find $f(x)$ such that

x	0	5	9
$f(x)$	1	31	91

Q2 Construct divided difference table and by using Newton's divided difference formula. Find the value of $f(8)$ from

x :	4	5	7	10	11	13
$f(x)$:	48	100	294	900	1210	2028

Q3 Find the lowest degree Polynomial for the following

x	-2	1	2	3
$f(x)$	-21	15	12	3

Reference Books

1. H.K. Dass – Advanced Engineering Mathematics – S. chand & Co., 9th Revised Ed.
2. Gupta and S. C. Boss – Introduction to Numerical Analysis – Academic Press Kolkata