## 5. SIMPLE MACHINES

## SIMPLE MACHINE:

A simple machine may be defined as a device, which enables us to do some useful work at some point or to overcome some resistance, when an effort or force is applied to it, at some other convenient point.

## COMPOUND MACHINE:

A compound machine may be defined as a device, consisting of a number of simple machines, which enables us to do some useful work at a faster speed or with a much less effort as compared to a simple machine.

## LIFTING MACHINE:

It is a device, which enables us to lift a heavy load $(W)$ by applying a comparatively smaller effort ( $P$ ).

## MECHANICAL ADVANTAGE:

The mechanical advantage (briefly written as M.A.) is the ratio of weight lifted ( $W$ ) to the effort applied $(P)$ and is always expressed in pure number. Mathematically, mechanical advantage, M.A. $=W / P$

## INPUT OF A MACHINE:

The input of a machine is the work done on the machine. In a lifting machine, it is measured by the product of effort and the distance through which it has moved.

## OUTPUT OF A MACHINE:

The output of a machine is the actual work done by the machine. In a lifting machine, it is measured by the product of the weight lifted and the distance through which it has been lifted.

## EFFICIENCY OF A MACHINE:

It is the ratio of output to the input of a machine and is generally expressed as a percentage. Mathematically, efficiency

$$
\eta=\frac{\text { Output }}{\text { Input }} \times 100
$$

## IDEAL MACHINE:

If the efficiency of a machine is $100 \%$ i.e., if the output is equal to the input, the machine is called as a perfect or an ideal machine.

## VELOCITY RATIO:

The velocity ratio (briefly written as V.R.) is the ratio of distance moved by the effort ( $y$ ) to the distance moved by the load ( $x$ ) and is always expressed in pure number. Mathematically, velocity ratio,

$$
\mathrm{V} . \mathrm{R} .=\frac{y}{x}
$$

## RELATION BETWEEN EFFICIENCY, MECHANICAL ADVANTAGE AND VELOCITY RATIO OF A LIFTING MACHINE:

Let
$W=$ Load lifted by the machine,
$P=$ Effort required to lift the load,
$Y=$ Distance moved by the effort, in lifting the load, and
$x=$ Distance moved by the load.
We know that

$$
\text { M.A. }=\frac{W}{P}=W / P \quad \text { and } \quad \text { V.R. }=\frac{y}{x}=y / x
$$

We also know that input of a machine

$$
\begin{align*}
& =\text { Effort applied } \times \text { Distance through which the effort has moved } \\
& =P \times y  \tag{i}\\
\text { and output of a machine } \quad & =\text { Load lifted } \times \text { Distance through which the load has been lifted } \\
& =W \times x \tag{ii}
\end{align*}
$$

$$
\therefore \text { Efficiency, } \quad \eta=\frac{\text { Output }}{\text { Input }}=\frac{W \times x}{P \times y}=\frac{W / P}{y / x}=\frac{\text { M.A. }}{\text { V.R. }}
$$

Note. It may be seen from the above relation that the values of M.A. and V.R. are equal only in case of a machine whose efficiency is $100 \%$. But in actual practice, it is not possible.

EXAMPLE: In a certain weight lifting machine, a weight of 1 kN is lifted by an effort of 25 N . While the weight moves up by 100 mm , the point of application of effort moves by 8 m . Find mechanical advantage, velocity ratio and efficiency of the machine.

Solution. Given: Weight $(W)=1 \mathrm{kN}=1000 \mathrm{~N}$; Effort $(P)=25 \mathrm{~N}$; Distance through which the weight is moved $(x)=100 \mathrm{~mm}=0.1 \mathrm{~m}$ and distance through which effort is moved $(y)=8 \mathrm{~m}$. Mechanical advantage of the machine.

We know that mechanical advantage of the machine

$$
\text { M.A. }=\frac{W}{P}=\frac{1000}{25}=40 \quad \text { Ans. }
$$

Velocity ratio of the machine
We know that velocity ratio of the machine

$$
\text { V.R. }=\frac{y}{x}=\frac{8}{0.1}=80 \quad \text { Ans. }
$$

## Efficiency of the machine

We also know that efficiency of the machine,

$$
\eta=\frac{M . A}{V \cdot R .}=\frac{40}{80}=0.5=50 \%
$$

Ans.

## REVERSIBILITY OF A MACHINE:

Sometimes, a machine is also capable of doing some work in the reversed direction, after the effort is removed. Such a machine is called a reversible machine and its action is known as reversibility of the machine.

## CONDITION FOR THE REVERSIBILITY OF A MACHINE:

Consider a reversible machine, whose condition for the reversibility is required to be found out.
Let $W=$ Load lifted by the machine,
$P=$ Effort required to lift the load,
$y=$ Distance moved by the effort, and
$x=$ Distance moved by the load
We know that input of the machine

$$
\begin{equation*}
=P \times y \tag{i}
\end{equation*}
$$

and output of the machine $\quad=W \times x$
We also know that machine friction

$$
\begin{equation*}
=\text { Input }- \text { Output }=(P \times y)-(W \times x) \tag{iii}
\end{equation*}
$$

A little consideration will show that in a reversible machine, the *output of the machine should be more than the machine friction, when the effort $(P)$ is zero. i.e.,

$$
W \times x>P \times y-W \times x
$$

or $\quad 2 W \times x>P \times y$
or $\quad \frac{W \times x}{P \times y}>\frac{1}{2}$
or $\quad \frac{\frac{W}{P}}{\frac{y}{x}}>\frac{1}{2}$
or $\quad \frac{\text { M.A }}{\text { V.R. }}>\frac{1}{2}$

$$
\ldots\left(\because \frac{W}{P}=\text { M.A. } \quad \text { and } \quad \frac{y}{x}=\text { V.R. }\right)
$$

$\therefore \quad \eta>\frac{1}{2}=0.5=50 \%$
Hence the condition for a machine, to be reversible,
is that its efficiency should be more than $50 \%$.

## SELF-LOCKING MACHINE:

Sometimes, a machine is not capable of doing any work in the reversed direction, after the effort is removed. Such a machine is called a non-reversible or self-locking machine. A little consideration will show, that the condition for a machine to be non-reversible or self-locking is that its efficiency should not be more than $50 \%$.

EXAMPLE: In a lifting machine, whose velocity ratio is 50 , an effort of 100 N is required to lift a load of 4 kN . Is the machine reversible ? If so, what effort should be applied, so that the machine is at the point of reversing?

Solution. Given: Velocity ratio (V.R.) $=50$; Effort $(P)=100 \mathrm{~N}$ and load $(W)=4 \mathrm{kN}=4000 \mathrm{~N}$. Reversibility of the machine

We know that M.A. $=\frac{W}{P}=\frac{4000}{100}=40$
and efficiency, $\quad \eta=\frac{\text { M.A. }}{\text { V.R. }}=\frac{40}{50}=0.8=80 \%$
Since efficiency of the machine is more than $50 \%$, therefore the machine is reversible. Ans. Effort to be applied

A little consideration will show that the machine will be at the point of reversing, when its efficiency is $50 \%$ or 0.5 .

Let
$P_{1}=$ Effort required to lift a load of 4000 N when the machine is at the point of reversing.

$$
\begin{array}{rlrl}
\text { We know that } & \text { M.A. } & =\frac{W}{P_{1}}=\frac{4000}{P_{1}}=4000 / P_{1} \\
\text { and efficiency, } & 0.5=\frac{\text { M.A }}{\text { V.R. }}=\frac{4000 / P_{1}}{50}=\frac{80}{P_{1}} \\
\therefore & P_{1}=\frac{80}{0.5}=160 \mathrm{~N} \quad \text { Ans. }
\end{array}
$$

LAW OF A MACHINE: The term 'law of a machine' may be defined as relationship between the effort applied and the load lifted. Thus for any machine, if we record the various efforts required to raise the corresponding loads, and plot a graph between effort and load, we shall get a straight line $A B$ as shown in Fig.


We also know that the intercept $O A$ represents the amount of friction offered by the machine. Or in other words, this is the effort required by the machine to overcome the friction, before it can lift any load.
Mathematically, the law of a lifting machine is given by
the relation:
P = mW + C
where $P=$ Effort applied to lift the load, $m=A$ constant (called coefficient of friction) which is equal to the slope of the line $A B$
W = Load lifted, and
$\mathrm{C}=$ Another constant, which represents the machine friction, (i.e. OA)

EXAMPLE: What load can be lifted by an effort of 120 N , if the velocity ratio is 18 and efficiency of the machine at this load is $60 \%$ ?
Determine the law of the machine, if it is observed that an effort of 200 N is required to lift a load of 2600 N and find the effort required to run the machine at a load of 3.5 kN .

Solution. Given: Effort $(P)=120 \mathrm{~N}$; Velocity ratio (VR. $)=18$ and efficiency $(\eta)=60 \%=0.6$. Load lifted by the machine.

Let $\quad W=$ Load lifted by the machine.
We know that M.A. $=\frac{W}{P}=\frac{W}{120}=W / 120$
and efficiency. $\quad 0.6=\frac{\text { M.A. }}{\text { V.R. }}=\frac{W / 120}{18}=\frac{W}{2160}$
$\therefore \quad W=0.6 \times 2160=1296 \mathrm{~N} \quad$ Ans.
Law of the machine
In the second case, $P=200 \mathrm{~N}$ and $W=2600 \mathrm{~N}$
Substituting the two values of $P$ and $W$ in the law of the machine, i.e., $P=m W+C$,

$$
\begin{align*}
120 & =m \times 1296+C  \tag{i}\\
200 & =m \times 2600+C \tag{ii}
\end{align*}
$$

Subtracting equation (i) from (ii),

$$
80=1304 m \quad \text { or } \quad m=\frac{80}{1304}=0.06
$$

and now substituting the value of $m$ in equation (ii)

$$
\begin{aligned}
200 & =(0.06 \times 2600)+C=156+C \\
C & =200-156=44
\end{aligned}
$$

Now substituting the value of $m=0.06$ and $C=44$ in the law of the machine,

$$
P=0.06 \mathrm{~W}+44 \quad \text { Ans. }
$$

Effort required to run the machine at a load of 3.5 kN .
Substituting the value of $W=3.5 \mathrm{kN}$ or 3500 N in the law of machine,

$$
P=(0.06 \times 3500)+44=254 \mathrm{~N} \quad \text { Ans. }
$$

## MAXIMUM MECHANICAL ADVANTAGE OF A LIFTING MACHINE:

We know that mechanical advantage of a lifting machine,

$$
\text { M.A. }=\frac{W}{P}
$$

For maximum mechanical advantage, substituting the value of $P=m W+C$ in the above equation.

$$
\text { Max. M.A. }=\frac{W}{m W+C}=\frac{1}{m+\frac{C}{W}}=\frac{1}{m}
$$

$$
\ldots\left(\text { Neglecting } \frac{C}{W}\right)
$$

## MAXIMUM EFFICIENCY OF A LIFTING MACHINE:

We know that efficiency of a lifting machine,

$$
\eta=\frac{\text { Mechanical advantage }}{\text { Velocity ratio }}=\frac{\frac{W}{P}}{\text { V.R. }}=\frac{W}{P \times V . R}
$$

For "maximum efficiency, substituting the value of $P=m W+C$ in the above equation,

$$
\operatorname{Max} . \eta=\frac{W}{(m W+C) \times \mathrm{V} \cdot \mathrm{R}}=\frac{1}{\left(m+\frac{C}{W}\right) \times \mathrm{V} \cdot \mathrm{R} .}=\frac{1}{m \times \mathrm{V} \cdot \mathrm{R} .} \ldots\left(\text { Neglecting } \frac{C}{W}\right)
$$

## SIMPLE GEAR TRAIN:



Now consider a simple train of wheels with one intermediate wheel as shown in Fig.
Let
N1 = Speed of the driver
T1 = No. of teeth on the driver,
N2, T2 = Corresponding values for the intermediate wheel, and
N3, T3 = Corresponding values for the follower.
Since the driver gears with the intermediate wheel, therefore

$$
\begin{equation*}
\frac{N_{2}}{N_{1}}=\frac{T_{1}}{T_{2}} \tag{I}
\end{equation*}
$$

Similarly, as the intermediate wheel gears with the follower, therefore
$\frac{N_{3}}{N_{2}}=\frac{T_{2}}{T_{3}}$
Multiplying equation (ii) by (i),
or

$$
\begin{aligned}
\frac{N_{3}}{N_{2}} \times \frac{N_{2}}{N_{1}} & =\frac{T_{2}}{T_{3}} \times \frac{T_{1}}{T_{2}} \\
\frac{N_{3}}{N_{1}} & =\frac{T_{1}}{T_{3}}
\end{aligned}
$$

$\therefore \quad \frac{\text { Speed of the follower }}{\text { Speed of the driver }}=\frac{\text { No. of teeth on the driver }}{\text { No. of teeth on the follower }}$

## COMPOUND GEAR TRAIN:



Let

$$
N_{1}=\text { Speed of the driver } 1
$$

$$
T_{1}=\text { No. of teeth on the driver } 1
$$

Similarly

$$
\begin{aligned}
N_{2}, N_{3}, \ldots N_{6} & =\text { Speed of the respective wheels } \\
T_{2}, T_{3}, \ldots T_{6} & =\text { No. of teeth on the respective wheels. }
\end{aligned}
$$

Since the wheel 1 gears with the wheel 2 , therefore

$$
\begin{equation*}
\frac{N_{2}}{N_{1}}=\frac{T_{1}}{T_{2}} \tag{i}
\end{equation*}
$$

Similarly

$$
\begin{equation*}
\frac{N_{4}}{N_{3}}=\frac{T_{3}}{T_{4}} \tag{ii}
\end{equation*}
$$

$$
\begin{equation*}
\frac{N_{6}}{N_{5}}=\frac{T_{5}}{T_{6}} \tag{iii}
\end{equation*}
$$

Multiplying equations (i), (ii) and (iii),

$$
\begin{aligned}
\frac{N_{2}}{N_{1}} \times \frac{N_{4}}{N_{3}} \times \frac{N_{6}}{N_{5}} & =\frac{T_{1}}{T_{2}} \times \frac{T_{3}}{T_{4}} \times \frac{T_{5}}{T_{6}} \\
\therefore \quad \frac{N_{6}}{N_{1}} & =\frac{T_{1} \times T_{3} \times T_{5}}{T_{2} \times T_{4} \times T_{6}} \quad\left(\because N_{2}\right. \\
& =\frac{\text { Product of the teeth on the drivers }}{\text { Product of the teeth on the followers }}
\end{aligned}
$$

## SIMPLE LIFTING MACHINES:

## 1. SIMPLE WHEEL AND AXLE:



In Fig. is shown a simple wheel and axle, in which the wheel A and axle B are keyed to the same shaft. The shaft is mounted on ball bearings, order to reduce the frictional resistance to a minimum. A string is wound round the axle $B$, which carries the load to be lifted. A second string is wound round the wheel $A$ in the opposite direction to that of the string on B.

Let $\mathrm{D}=$ Diameter of effort wheel, d = Diameter of the load axle,
W = Load lifted, and $P=$ Effort applied to lift the load.

One end of the string is fixed to the wheel, while the other is free and the effort is applied to this end. Since the two strings are wound in opposite directions, therefore a downward motion of the effort $(P)$ will raise the load $(W)$.

Since the wheel as well as the axle are keyed to the same shaft, therefore when the wheel rotates through one revolution, the axle will also rotate through one revolution. We know that displacement of the effort in one revolution of effort wheel A,

$$
=\pi D \ldots(i)
$$

and displacement of the load in one revolution

$$
=\pi d . . .(i i)
$$

$$
\begin{array}{ll}
\therefore & \text { V.R. }=\frac{\text { Distance moved by the effort }}{\text { Distance moved by the load }}=\frac{\pi D}{\pi d}=\frac{D}{d} \\
\text { Now } & \text { M.A }=\frac{\text { Load lifted }}{\text { Effort applied }}=\frac{W}{P} \\
\text { and efficiency } & \eta=\frac{\text { M.A }}{\text { V.R. }}
\end{array}
$$

EXAMPLE: A simple wheel and axle has wheel and axle of diameters of 300 mm and 30 mm respectively. What is the efficiency of the machine, if it can lift a load of 900 N by an effort of 100 N .

Solution. Given: Diameter of wheel $(D)=300 \mathrm{~mm}$; Diameter of axle $($ d $)=30 \mathrm{~mm}$; Load lifted by the machine $(W)=900 \mathrm{~N}$ and effort applied to lift the load $(P)=100 \mathrm{~N}$

We know that velocity ratio of the simple wheel and axle,

$$
\mathrm{V} . \mathrm{R} .=\frac{D}{d}=\frac{300}{30}=10
$$

and mechanical advantage

$$
\text { M.A. }=\frac{W}{P}=\frac{900}{100}=9
$$

$$
\therefore \quad \text { Efficiency, } \eta=\frac{\text { M.A. }}{\text { V.R. }}=\frac{9}{10}=0.9=90 \% \quad \text { Ans. }
$$

## 2. SINGLE PURCHASE CRAB WINCH:



In single purchase crab winch, a rope is fixed to the drum and is wound a few turns round it. The free end of the rope carries the load $W$. A toothed wheel $A$ is rigidly mounted on the load drum. Another toothed wheel $B$, called pinion, is geared with the toothed wheel $A$ as shown in Fig. The effort is applied at the end of the handle to rotate it.

Let $\quad T_{1}=$ No. of teeth on the main gear (or spur wheel) $A$,
$T_{2}=$ No. of teeth on the pinion $B$,
$l=$ Length of the handle,
$r=$ Radius of the load drum.
$W=$ Load lifted, and
$P=$ Effort applied to lift the load.
We know that distance moved by the effort in one revolution of the handle,

$$
\begin{equation*}
=2 \pi l \tag{i}
\end{equation*}
$$

$\therefore$ No. of revolutions made by the pinion $B$

$$
=1
$$

and no. of revolutions made by the wheel $A$

$$
=\frac{T_{2}}{T_{1}}
$$

$\therefore$ No. of revolutions made by the load drum

$$
\begin{array}{ll} 
& =\frac{T_{2}}{T_{1}} \\
\text { and distance moved by the load } & =2 \pi r \times \frac{T_{2}}{T_{1}}  \tag{ii}\\
\begin{array}{rlrl}
\therefore & \text { V.R. } & =\frac{\text { Distance moved by the effort }}{\text { Distance moved by the load }}=\frac{2 \pi l}{2 \pi r \times \frac{T_{2}}{T_{1}}}=\frac{l}{r} \times \frac{T_{1}}{T_{2}} \\
\text { Now } & \text { M.A. } & =\frac{W}{P} \\
\text { and efficiency. } & \eta & =\frac{\text { M.A. }}{\text { V.R. }} & \ldots \text { as usual }
\end{array} \\
&
\end{array}
$$

EXAMPLE: In a single purchase crab winch, the number of teeth on pinion is 25 and that on the spur wheel 100. Radii of the drum and handle are 50 mm and 300 mm respectively. Find the efficiency of the machine and the effect of friction, if an effort of 20 N can lift a load of 300 N .

Solution. Given: No. of teeth on pinion $\left(T_{2}\right)=25$; No. of teeth on the spur wheel $\left(T_{1}\right)=100$; Radius of daum $(r)=50 \mathrm{~mm}$; Radius of the handle or length of the handle $(l)=300 \mathrm{~mm}$; Effort $(P)=20$ N and load lifted $(W)=300 \mathrm{~N}$.
Efficiency of the machine
We know that velocity ratio
and

$$
\begin{aligned}
& \text { V.R. }=\frac{1}{r} \times \frac{T_{1}}{T_{2}}=\frac{300}{50} \times \frac{100}{25}=24 \\
& \text { M.A }=\frac{W}{P}=\frac{300}{20}=15
\end{aligned}
$$

$$
\therefore \text { Efficiency. } \quad \eta=\frac{\text { M.A. }}{\text { V.R. }}=\frac{15}{24}=0.625=62.5 \%
$$

Ans.
Effect of friction
We know that effect of friction in terms of load,

$$
F_{(\text {load })}=(P \times \text { V.R. })-W=(20 \times 24)-300=180 \mathrm{~N}
$$

and effect of friction in terms of effort,

$$
F_{\text {(effort) }}=P-\frac{W}{\mathrm{~V} \cdot \mathrm{R} .}=20-\frac{300}{24}=7.5 \mathrm{~N}
$$

It means that if the machine would have been ideal (i.e. without friction) then it could lift an extra load of 180 N with the same effort of 20 N . Or it could have required 7.5 N less force to lift the same load of 300 N . Ans.

## EXAMPLE:

A single purchase crab winch, has the following details:
Length of lever $\quad=700 \mathrm{~mm}$
Number of pinion teeth $=12$
Number of spur gear teeth $=96$
Diameter of load axle $=200 \mathrm{~mm}$
It is observed that an effort of 60 N can lift a load of 1800 N and an effort of 120 N can lift a load of 3960 N .
What is the law of the machine? Also find efficiency of the machine in both the cases.
Solution. Given: Length of lever $(l)=700 \mathrm{~mm}$; No. of pinion teeth $\left(T_{2}\right)=12$; No. of spur geer teeth $\left(T_{1}\right)=96$ and dia of load axle $=200 \mathrm{~mm}$ or radius $(r)=200 / 2=100 \mathrm{~mm}$.
(i) Law of the machine

When $P_{1}=60 \mathrm{~N}, W_{1}=1800 \mathrm{~N}$ and when $P_{2}=120 \mathrm{~N}, W_{2}=3960 \mathrm{~N}$.
Substituting the values of $P$ and $W$ in the law of the machine i.e., $P=m W+C$
and

$$
\begin{align*}
60 & =(m \times 1800)+C  \tag{i}\\
120 & =(m \times 3960)+C \tag{ii}
\end{align*}
$$

Subtracting equation (i) from equation (ii)

$$
60=m \times 2160
$$

or

$$
m=\frac{60}{2160}=\frac{1}{36}
$$

Now substituting this value of $m$ in equation ( $i$ ),

$$
\begin{array}{ll} 
& 60=\left(\frac{1}{36} \times 1800\right)+C=50+C \\
\therefore & C=60-50=10
\end{array}
$$

and now substituting the value of $m=1 / 36$ and $C=10$ in the law of machine,

$$
P=\frac{1}{36} W+10
$$

Ans.
(ii) Efficiencies of the machine in both the cases

We know that velocity ratio

$$
\text { V.R. }=\frac{l}{r} \times \frac{T_{1}}{T_{2}}=\frac{700}{100} \times \frac{96}{12}=56
$$

and mechanical advantage in the first case

$$
\text { M.A. }=\frac{W_{1}}{P_{1}}=\frac{1800}{60}=30
$$

$\therefore$ Efficiency

$$
\eta_{1}=\frac{\text { M.A }}{\text { V.R. }}=\frac{30}{56}=0.536=53.6 \%
$$

Ans.

Similarly, mechanical advangate in the second case,

$$
\text { M.A. }=\frac{W_{2}}{P_{2}}=\frac{3960}{120}=33
$$

$\therefore$ Efficiency

$$
\eta_{2}=\frac{M . A}{V . R}=\frac{33}{56}=0.589=58.9 \%
$$

Ans.

## DOUBLE PURCHASE CRAB WINCH:

A double purchase crab winch is an improved form of a single purchase crab winch, in which the velocity ratio is intensified with the help of one more spur wheel and a pinion. In a double purchase crab winch, there are two spur wheels of teeth $T 1$ and $T 2$ and $T 3$ as well as two pinions of teeth $T 2$ and $T 4$.


The arrangement of spur wheels and pinions are such that the spur wheel with $T 1$ gears with the pinion of teeth $T 2$. Similarly, the spur wheel with teeth $T 3$ gears with the pinion of the teeth $T 4$, The effort is applied to a handle as shown in Fig.

Let

$$
\begin{aligned}
T_{1} \text { and } T_{3} & =\text { No. of teeth of spur wheels, } \\
T_{2} \text { and } T_{4} & =\text { No. of teeth of the pinions } \\
l & =\text { Length of the handle. } \\
r & =\text { Radius of the load drum, } \\
W & =\text { Load lifted, and } \\
P & =\text { Effort applied to lift the load, at the end of the handle. }
\end{aligned}
$$

We know that distance moved by the effort in one revolution of the handle,

$$
\begin{equation*}
=2 \pi l \tag{i}
\end{equation*}
$$

$\therefore$ No. of revolutions made by the pinion 4

$$
=1
$$

and no. of revolutions made by the wheel 3

$$
=\frac{T_{4}}{T_{3}}
$$

$\therefore$ No. of revolutions made by the pinion 2

$$
=\frac{T_{4}}{T_{3}}
$$

and no. of revolutions made by the wheel 1

$$
=\frac{T_{2}}{T_{1}} \times \frac{T_{4}}{T_{3}}
$$

$\therefore$ Distance moved by the load

$$
\begin{equation*}
=2 \pi r \times \frac{T_{2}}{T_{1}} \times \frac{T_{4}}{T_{3}} \tag{iii}
\end{equation*}
$$

$\therefore \quad$ V.R. $=\frac{\text { Distance moved by the effort }}{\text { Distance moved by the load }}$

$$
=\frac{2 \pi l}{2 \pi r \times \frac{T_{2}}{T_{1}} \times \frac{T_{4}}{T_{3}}}=\frac{l}{r}\left(\frac{T_{1}}{T_{2}} \times \frac{T_{3}}{T_{4}}\right)
$$

Now
and efficiency,

$$
\begin{aligned}
\text { M.A. } & =\frac{W}{P} \\
\eta & =\frac{M \cdot A}{\text { V.R. }}
\end{aligned}
$$

## EXAMPLE:

In a double purchase crab winch, teeth of pinions are 20 and 25 and that of spur wheels are 50 and 60 . Length of the handle is 0.5 metre and radius of the load drum is 0.25 metre. If efficiency of the machine is $60 \%$, find the effort required to lift a load of 720 N .

Solution. Given: No. of teeth of pinion $\left(T_{2}\right)=20$ and $\left(T_{4}\right)=25$; No. of teeth of spur wheel $\left(T_{1}\right)=501$ and $\left(T_{3}\right)=60$; Length of the handle $(l)=0.5 \mathrm{~m}$; Radius of the load drum $(r)=0.25$ m; Efficiency $(\eta)=60 \%=0.6$ and load to be lifted $(W)=720 \mathrm{~N}$.

Let
$P=$ Effort required in newton to lift the load.
We know that velocity ratio

$$
\text { V.R. }=\frac{1}{r}\left(\frac{T_{1}}{T_{2}} \times \frac{T_{3}}{T_{4}}\right)=\frac{0.5}{0.25}\left(\frac{50}{20} \times \frac{60}{25}\right)=12
$$

and

$$
\text { M.A. }=\frac{W}{P}=\frac{720}{P}
$$

$\therefore$ Efficiency
or

$$
0.6=\frac{\text { M.A. }}{\text { V.R. }}=\frac{\frac{720}{P}}{12}=\frac{60}{P}
$$

$$
P=\frac{60}{0.6}=100 \mathrm{~N} \quad \text { Ans. }
$$

WORM AND WORM WHEEL:


It consists of a square threaded screw, $S$ (known as worm) and a toothed wheel (known as worm wheel) geared with each other, as shown in Fig. A wheel $A$ is attached to the worm, over which passes a rope as shown in the figure. Sometimes, a handle is also fixed to the worm (instead of the wheel). A load drum is securely mounted on the worm wheel.

Let;
$D=$ Diameter of the effort wheel,
$r=$ Radius of the load drum
W = Load lifted,
P = Effort applied to lift the load, and
$\mathrm{T}=$ No. of teeth on the worm wheel.
We know that distance moved by the effort in one revolution of the wheel (or handle)

$$
=\pi D \ldots(i)
$$

If the worm is single-threaded (i.e., for one revolution of the wheel $A$, the screw $S$ pushes the worm wheel through one teeth), then the load drum will move through

$$
=\frac{1}{T} \text { revolution }
$$

and distance, through which the load will move

$$
\begin{align*}
& =\frac{2 \pi r}{T}  \tag{il}\\
\therefore \quad \text { V.R. } & =\frac{\text { Distance moved by the effort }}{\text { Distance moved by the load }} \\
& =\frac{\pi D}{\frac{2 \pi r}{T}}=\frac{D T}{2 r} \tag{ili}
\end{align*}
$$

Now
and efficiency,

$$
\begin{aligned}
\text { M.A. } & =\frac{W}{P} \\
\eta & =\frac{\text { M.A. }}{\text { V.R. }}
\end{aligned}
$$

as usual
...as usual

Notes: 1. If the worm is double-threaded i.e., for one revolution of wheel $A$, the screw $S$ pushes the worm wheel through two teeths, then

$$
\mathrm{V} . \mathrm{R} .=\frac{D T}{2 \times 2 r}=\frac{D T}{4 r}
$$

2. In general, if the worm is $n$ threaded, then

$$
\mathrm{V} \cdot \mathrm{R}=\frac{D T}{2 n r}
$$

EXAMPLE: A worm and worm wheel with 40 teeth on the worm wheel has effort wheel of 300 mm diameter and load drum of 100 mm diameter. Find the efficiency of the machine, if it can lift a load of 1800 N with an effort of 24 N .

Solution. Given: No. of teeth on the worm wheel $(T)=40$; Diameter of effort wheel $=300 \mathrm{~mm}$ Diameter of load drum $=100 \mathrm{~mm}$ or radius $(r)=50 \mathrm{~mm}$; Load lifted $(W) 1800 \mathrm{~N}$ and effort $(P)=24 \mathrm{~N}$.

We know that velocity ratio of worm and worm wheel,

$$
\mathrm{V.R}=\frac{D T}{2 r}=\frac{300 \times 40}{2 \times 50}=120
$$

and

$$
\text { M.A. }=\frac{W}{P}=\frac{1800}{24}=75
$$

$\therefore$ Efficiency, $\quad \eta=\frac{\text { M.A. }}{\text { V.R. }}=\frac{75}{120}=0.625=62.5 \%$
Ans.

## SCREW JACK:

It consists of a screw, fitted in a nut, which forms the body of the jack. The principle, on which a screw jack works, is similar to that of an inclined plane.


Fig. shows a simple screw jack, which is rotated by the application of an effort at the end of the lever, for lifting the load. Now consider a single threaded simple screw jack.

Let

$$
\begin{aligned}
l & =\text { Length of the effort arm, } \\
p & =\text { Pitch of the screw, } \\
W & =\text { Load lifted, and } \\
P & =\text { Effort applied to lift the load at the end of teh lever. }
\end{aligned}
$$

We know that distance moved by the effort in one revolution of screw,

$$
\begin{equation*}
=2 \pi l \tag{i}
\end{equation*}
$$

and distance moved by the load $=p$
$\therefore \quad$ Velocity ratio $=\frac{\text { Distance moved by the effort }}{\text { Distance moved by the load }}=\frac{2 \pi l}{p}$
Now
and efficeincy,

$$
\begin{align*}
\text { M.A. } & =\frac{W}{P}  \tag{iii}\\
\eta & =\frac{\text { M.A }}{\text { V.R. }}
\end{align*}
$$

...as usual

Note: The value of $P$ i.e., the effort applied may also found out by the relation :
where

$$
\text { where } \quad W=\text { Load lifted }
$$

and

$$
\begin{aligned}
* P & =W \tan (\alpha+\phi) \\
W & =\text { Load lifted } \\
\tan \alpha & =\frac{p}{\pi d} \\
\tan \phi & =\mu=\text { Coefficient of friction. }
\end{aligned}
$$

## EXAMPLE:

A screw jack has a thread of 10 mm pitch. What effort applied at the end of a handle 400 mm long will be required to lift a load of 2 kN , if the efficiency at this load is $45 \%$.

SOLUTION: Given: Pitch of thread $(p)=10 \mathrm{~mm}$;
Length of the handle $(I)=400 \mathrm{~mm}$;
Load lifted $(W)=2 \mathrm{kN}=2000 \mathrm{~N}$ and efficiency $(n)=45 \%=0.45$.
Let $P=$ Effort required to lift the load
We know that velocity ratio
and

$$
\begin{aligned}
& \mathrm{V} \cdot \mathrm{R} .=\frac{2 \pi l}{p}=\frac{2 \pi \times 400}{10}=251.3 \\
& \mathrm{M.A}=\frac{W}{P}=\frac{2000}{P}
\end{aligned}
$$

We also know that efficiency,

$$
\begin{aligned}
0.45 & =\frac{\text { M.A. }}{\text { V.R. }}=\frac{\frac{2000}{P}}{251.3}=\frac{7.96}{P} \\
P & =\frac{7.96}{0.45}=17.7 \mathrm{~N} \quad \text { Ans. }
\end{aligned}
$$

## HOISTING MACHINE:

Mechanisms for raising and lowering material with intermittent motion while holding the material freely suspended. Hoisting machines are capable of picking up loads at one location and depositing them at another anywhere within a limited area. In contrast, elevating machines move their loads only in a fixed vertical path, and monorails operate on a fixed horizontal path rather than over a limited area.

The principal components of hoisting machines are: sheaves and pulleys, for the hoisting mechanisms; winches and hoists, for the power units; and derricks and cranes, for the structural elements.

## TYPES:

- Pulley and sheave block
- Chain hoists
- Mobile cranes
- Winch
- Jack
- Shear leg
- Tower cranes
- Whirler cranes
- Derrick cranes
- Gantry cranes


## DERRICK:

A derrick is a lifting device composed at minimum of one guyed mast, as in a gin pole, which may be articulated over a load by adjusting its guys. Most derricks have at least two components, either a guyed mast or self-supporting tower, and a boom hinged at its base to provide articulation, as in a stiffleg derrick.

The most basic type of derrick is controlled by three or four lines connected to the top of the mast, which allow it both to move laterally and cant up and down. To lift a load, a separate line runs up and over the mast with a hook on its free end, as with a crane.

Forms of derricks are commonly found aboard ships and at docking facilities. Some large derricks are mounted on dedicated vessels, and known as floating derrick and sheerlegs.

The term derrick is also applied to the framework supporting a drilling apparatus in an oil rig.

