LECTURE 10

Polytropic Process

$$W = \int c\,dv/ v^n$$

$$w = (P_1v_1- P_2v_2)/(n-1)$$

$$du = dq – dw$$

$$u_2 – u_1 = q - (P_1v_1- P_2v_2)/(n-1)$$

$$u_2 – u_1 = C_v \ (T_2 – T_1) = q – w$$

$$q = R(T_2 – T_1)/(\gamma-1) + (P_1v_1- P_2v_2)/(n-1)$$

$$= R (T_1 – T_2)\{1/(n-1) – 1/(\gamma-1)\}$$

$$=(P_1v_1- P_2v_2)/(n-1) \{(\gamma -n)/(\gamma-1)\}$$

$$=w.\{ (\gamma -n)/(\gamma-1)\}$$

Problem: Air (ideal gas with $\gamma = 1.4$) at 1 bar and 300K is compressed till the final volume is one-sixteenth of the original volume, following a polytropic process $Pv^{1.25}$ = const. Calculate (a) the final pressure and temperature of the air, (b) the work done and (c) the energy transferred as heat per mole of the air.

Solution: (a) $P_1 v_1^{1.25} = P_2 v_2^{1.25}$

$P_2 = P_1(v_1/v_2)^{1.25} = 1(16)^{1.25} =$ **32 bar**

$T_2 = (T_1 P_2 v_2)/(P_1 v_1) = (300 \times 32 \times 1)/(1 \times 16)$

$=$ **600K**

(b) $w = (P_1 v_1 - P_2 v_2)/(n-1)$

$= R_u(T_1 - T_2)/(n-1)$

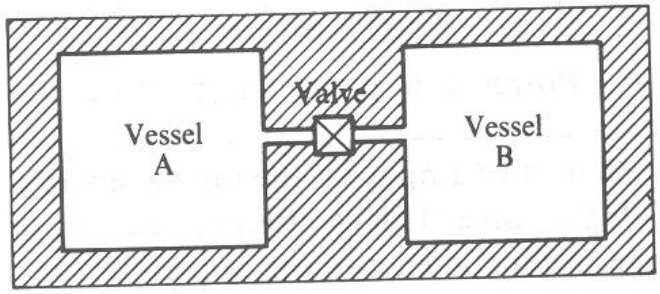
$= 8.314\ (300 - 600)/(1.25-1) =$ **-9.977 kJ/mol**

(c) $q = w.\{ (\gamma - n)/(\gamma-1)\}$

$= -9.977\ (1.4 - 1.25)/(1.4-1)$

$=$ **-3.742 kJ/mol**

Unresisted or Free expansion



In an irreversible process, $w \neq \int Pdv$

Vessel A: Filled with fluid at pressure

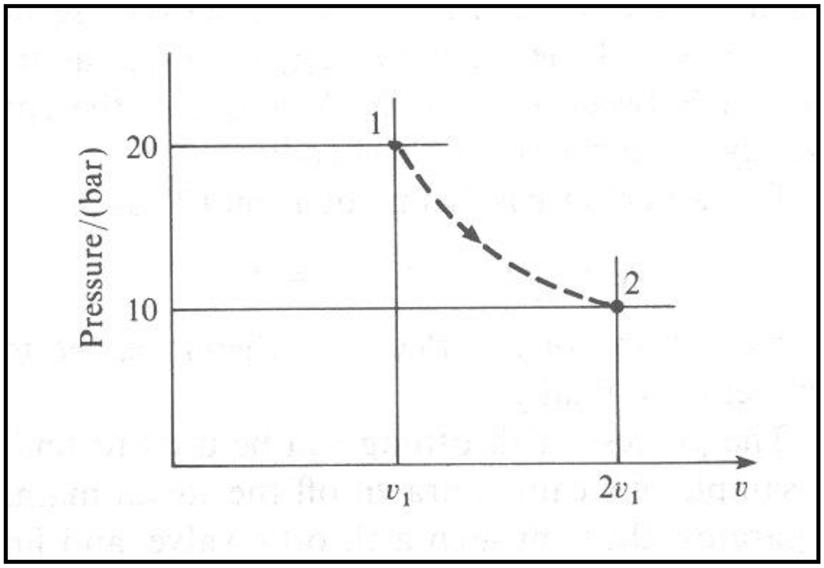
Vessel B: Evacuated/low pressure fluid

Valve is opened: Fluid in A expands and fills both vessels A and B. This is known as unresisted expansion or free expansion.

No work is done on or by the fluid.

No heat flows (Joule's experiment) from the boundaries as they are insulated.

$U_2 = U_1 \quad (U = U_A + U_B)$



Problem: A rigid and insulated container of $2m^3$ capacity is divided into two equal compartments by a membrane. One compartment contains helium at 200kPa and 127^oC while the second compartment contains nitrogen at 400kPa and 227^oC. The membrane is punctured and the gases are allowed to mix. Determine the temperature and pressure after equilibrium has been established. Consider helium and nitrogen as perfect gases with their C_v as 3R/2 and 5R/2 respectively.

Solution: Considering the gases contained in both the compartments as the system, W= 0 and Q = 0. Therefore, $\Delta U = 0$ ($U_2 = U_1$)

Amount of helium = $N_{He} = P_A V_A / R_u T_A$
$= 200 \times 10^3 \times 1/(8.314 \times 400)$
$= 60.14$ mol.

Amount of nitrogen = $N_{N2} = P_B V_B / R_u T_B$
$= 400 \times 10^3 \times 1/(8.314 \times 500)$
$= 96.22$ mol.

Let T_f be the final temperature after equilibrium has been established. Then,

$$[NC_v(T_f\text{-}400)]_{He} + [NC_v(T_f\text{-}500)]_{N2} = 0$$

$$R_u[60.14(T_f\text{-}400)3 + 96.22(T_f\text{-}500)5\ ]\ /2 = 0$$

Or, $\mathbf{T_f = 472.73\ K}$

The final pressure of the mixture can be obtained by applying the equation of state:

$$P_fV_f = (N_{He} + N_{N2})R_u\ T_f$$

$$2P_f = (60.14 + 96.22)\ 8.314\ (472.73)$$

or, $\mathbf{P_f = 307.27\ kPa}$