LECTURE 11

Control-Volume Analysis

Control volume is a volume in space of special interest for particular analysis.

The surface of the control volume is referred as a control surface and is a closed surface.

The surface is defined with relative to a coordinate system that may be fixed, moving or rotating.

Mass, heat and work can cross the control surface and mass and properties can change with time within the control volume.

Examples: turbines, compressors, nozzle, diffuser, pumps, heat exchanger, reactors, a thrust-producing device, and combinations of these.

First law of thermodynamics for a continuous system

Let the continuous system be in state 1 at time t and after a differential time dt, let it be in the state 2. The change in the energy of the continuous system is,

$$dE = \frac{d[\int \rho e dV]}{dt} dt$$

Now,

$$dE = dQ - dW$$

or,

$$\frac{dE}{dt} = \dot{Q} - \dot{W}$$

$$\frac{d}{dt} \int_V \rho e dV = \dot{Q} - \dot{W}$$

First law of thermodynamics to a control volume

$$\frac{dm}{dt} = \dot{m}_i - \dot{m}_e$$

Or

[Rate of accumulation of mass inside the control volume] = [Rate of mass entering the control volume at inlet] – [Rate of mass leaving the control volume at exit]

The above is commonly known as **continuity equation.**

We should identify a definite quantity of matter which remains constant as the matter flows. For this purpose, let the boundary of the system include all matter inside the control volume and that which is about to enter the control volume during the differential time interval dt.

At time t, **the system is defined as the mass contained in the control volume and the mass in region A which is about to enter the control volume in a differential time dt.**

At time t+dt, **the system is defined as the mass contained in the control volume and the mass in region B.**

Therefore, during the differential time dt, the system configuration undergoes a change.

Mass contained in region A = $\dot{m}_i\, dt$

Mass contained in region B = $\dot{m}_e\, dt$

From mass balance,

$$m(t) + \dot{m}_i\,dt = m(t+dt) + \dot{m}_e\,dt$$

The work done as the mass enters the control volume = $-P_i v_i \dot{m}_i\, dt$

The work done by mass exiting the control volume = $P_e v_e \dot{m}_e\, dt$

Energy of the system at time t = $E(t) + \dot{m}_i\, e_i dt$

Energy of the system at time (t+dt) =

$E(t+dt) + e_e\, \dot{m}_e\, dt$

Energy transferred as heat to the system = $\dot{Q}\,dt$

Shaft work done by the system = $\dot{W}_{sh}\,dt$

From the first law,

$$[E(t+dt) \quad \dot{m}_e \quad + \; e_e \; \dot{m}_e\, dt] - [E(t) + \dot{m}_i\, e_i dt\;] = \dot{Q}\,dt - \dot{W}_{sh}\,dt - (\qquad P_e v_e \dot{m}_e - P_i v_i \dot{m}_i\;)dt$$

$$\dot{m}_e(\, e_e + P_e v_e) - \dot{m}_i(e_i + P_i v_i) = \dot{Q} - \dot{W}_{sh} - \frac{E(t+dt)-E(t)}{dt}$$

or,

$$\dot{m}_e \quad (h_e + \mathbf{V}_e^{2}/2 + gZ_e) - \dot{m}_i(h_i + \mathbf{V}_i^{2}/2 + gZ_i) = \dot{Q} - \dot{W}_{sh} - dE/dt$$

where, $h_e = u_e + P_e v_e,\;\; h_i = u_i + P_i v_i$

Or, Rate of energy accumulation = rate of energy inflow – rate of energy outflow

Steady state flow process

Assumptions:

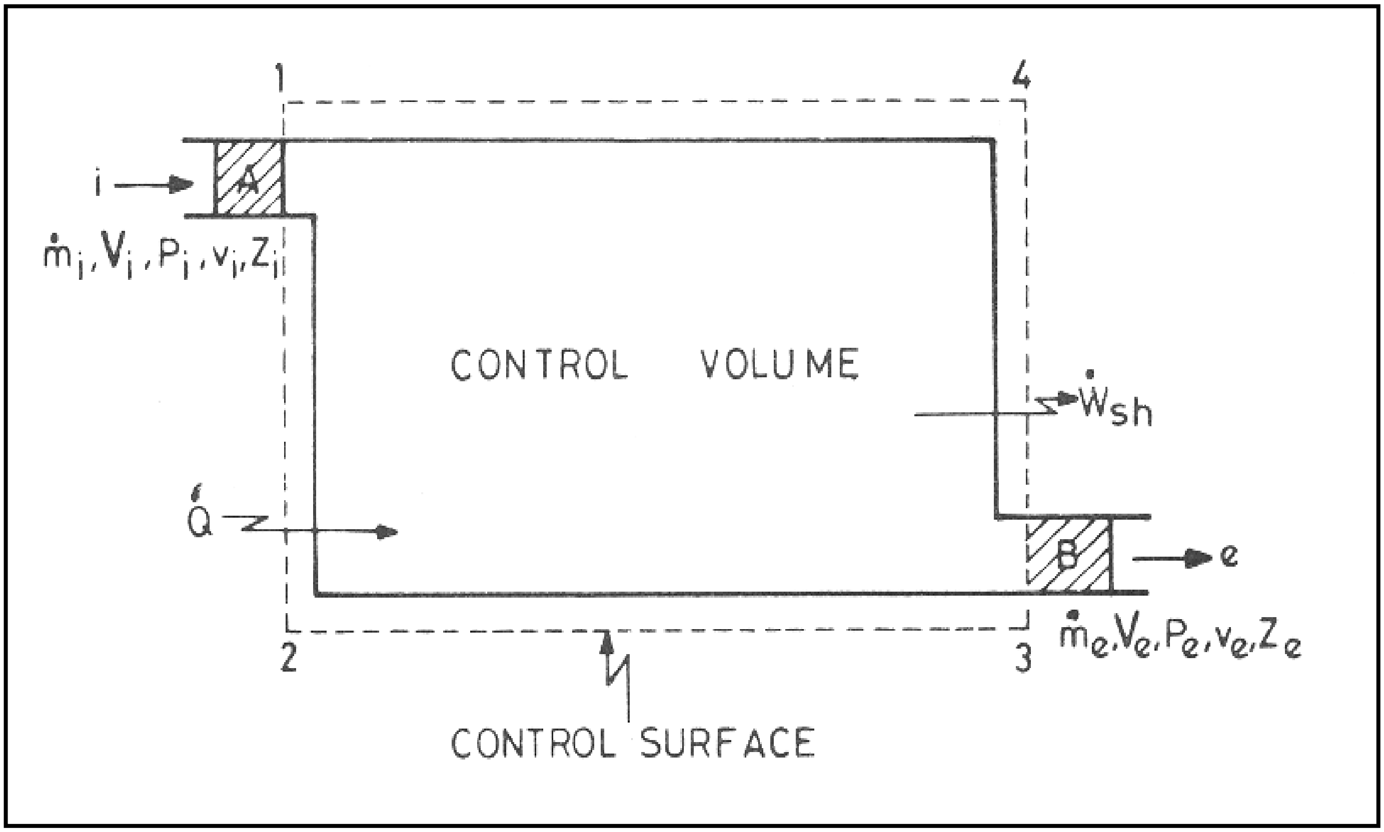
$\dot{m}_i = \dot{m}_e = \dot{m}$

The state of matter at the inlet, exit and at any given point inside the control volume does not change with respect to time.

$$dE / dt = 0$$

The rate of energy transfers across the control surface is constant.

$$(h_e+\boldsymbol{V}_e^2/2 + gZ_e) - (h_i+\boldsymbol{V}_i^2/2 + gZ_i) = (\dot{Q} - \dot{W}_{sh})/\dot{m}$$


1
4
i
A
$\dot{m}_i, V_i, P_i, v_i, Z_i$
CONTROL VOLUME
$\dot{W}_{sh}$
$\dot{Q}$
B
e
$\dot{m}_e, V_e, P_e, v_e, Z_e$
2
3
CONTROL SURFACE