LECTURE 13

A systematic approach to problem solving

Step 1. Identify the system and draw a sketch of it. The system that is about to be analyzed should be identified on the sketch by drawing its boundaries using the dashed lines.

Step 2. List the given information on the sketch. Heat and work interactions if any should also be indicated on the sketch with proper directions.

Step 3. State any assumptions:
The simplifying assumptions that are made to solve a problem should be stated and fully justified.
Commonly made assumptions:

- **(a)** Assuming process to be quasi-equilibrium
- **(b)** Neglecting *PE* and *KE*
- **(c)** Treating gas as ideal
- **(d)** Neglecting heat transfer from insulated systems.

Step 5. Apply the conservation equations.

Step 6. Draw a process diagram.

Determine the required properties and unknowns.

Problem # 1 A 0.1 m^3 rigid tank contains steam initially at 500 kPa and 200°C. The steam is now allowed to cool until the temperature drops to 50°C. Determine the amount of heat transfer during this process and the final pressure in the tank.

State 1: P_1 = 500kPa, T_1 = 200°C
$v_1 = 0.4249\ m^3/kg$, $u_1 = 2642.9$ kJ/kg

State 2: $v_2 = v_1 = 0.4269\ m^3/kg$
$T_2 = 50°C \rightarrow v_f = 0.001 m^3/kg$
$v_g = 12.03\ m^3/kg$
$u_f = 209.32$ kJ/kg
$u_g = 2443.5$ kJ/kg

$P_2 = P_{sat}$ @50°c = **12.349 kPa**

$v_2 = v_f + x_2 v_{fg}$

$0.4249 = 0.001 + x_2(12.03 = 0.001)$

$x_2 = 0.0352$

$u_2 = u_f \ + x_2 u_g$

$= 209.32 + (0.0352)(2443.5 - 209.32)$

$= 288.0$ kJ/kg

$m = V/u = (0.1 \text{ m}^3/\text{kg})/(0.4249 \text{ m}^3/\text{kg})$

$= 0.235$ kg

$-Q_{out} = \Delta U = m(u_2 - u_1)$

$Q_{out} = m(u_1 - u_2)$

$= (0.235)(2642.9 - 288)$

= **553.4 kg**

Problem # 2 A piston/cylinder contains 50 kg of water at 200 kPa with a volume of 0.1 m^3 . Stop in the cylinder is placed to restrict the enclosed volume to 0.5 m^3. The water is now heated until the piston reaches the stops. Find the necessary heat transfer.

At 200 kPa,
v_f = 0.001061 m^3/kg
v_{fg} = 0.88467 m^3/kg
h_f = 504.68 kJ/kg
h_{fg} = 2201.96 kJ/kg

This is a constant pressure process. Hence,
$Q = \Delta H$

The specific volume initially,

$$v_i = 0.1 /50 = 0.002 \text{ m}^3/\text{kg}$$

$v = v_f + x\, v_{fg}$
$\quad = 0.001061 + x\, (0.88467)$
Therefore, $x = (0.002 – 0.001061) / 0.88467$
$\quad\quad = 0.001061$

$h = h_f + x\, h_{fg}$
$\quad = 504.68 + 0.001061(2201.96)$
$\quad = 507.017$ kJ/kg

$\underline{v_{final}} = 0.5 / 50 = 0.01\ m^3/kg$

$v = v_f + x\, v_{fg}$

Therefore, $x = (0.01 – 0.001061) / 0.88467$
$\quad = 0.01$

$h_{final} = 504.68 + 0.01(2201.96)$
$\quad = 526.69$ kJ/kg

$Q = \Delta H = 50\ (526.69 - 507.017)$
$\quad =$ **983.65 kJ/kg**

Problem # 3 A rigid insulated tank is separated into two rooms by a stiff plate. Room A of 0.5 m^3 contains air at 250 kPa, 300 K and room B of 1 m^3 has air at 150 kPa, 1000 K. The plate is removed and the air comes to a uniform state without any heat transfer. Find the final pressure and temperature.

The system comprises of room A and B together. This is a constant internal energy process as there is no heat and work exchange with the surroundings.

$$m_A = P_A V_A / RT_A$$
$$= (250 \text{ x } 1000 \text{ x } 0.5) / (287 \text{ x } 300)$$
$$= 1.452 \text{ kg}$$
$$m_B = P_B V_B / RT_B$$
$$= (150 \text{ x } 1000 \text{ x } 1.0) / (287 \text{ x } 1000)$$
$$= 0.523 \text{ kg}$$

$$\Delta U_A + \Delta U_B = 0$$

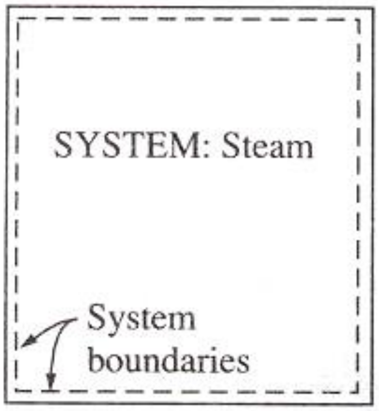
Let T_f be the final temperature at equilibrium

$$m_A\,(T_f - 300) + m_B\,(T_f - 1000) = 0$$

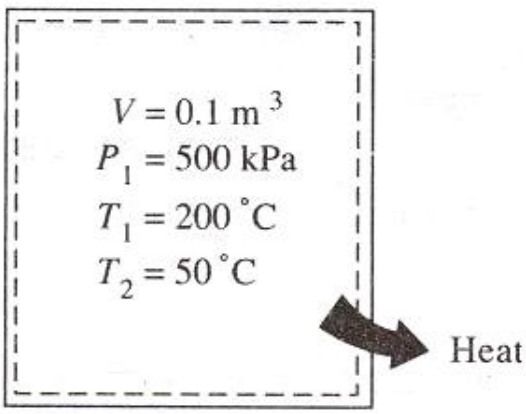
$$1.452\,(T_f - 300) + 0.523\,(T_f - 1000) = 0$$

T_f = **<u>485.37 K</u>**

P_f = (1.452 + 0.523) x 287 x 485.37 / 1.5
= **<u>183.41 kPa</u>**



Step 1: Draw a sketch of the system and system boundaries.



Step 2: List the given information on the sketch.

$v_1 = v_2$

$W = 0$

Step 3: Look for simplifications.

Step 4: Make realistic assumptions, if necessary.

Assume:

(1) $\Delta PE = 0$ (since there is no mention of elevation change).

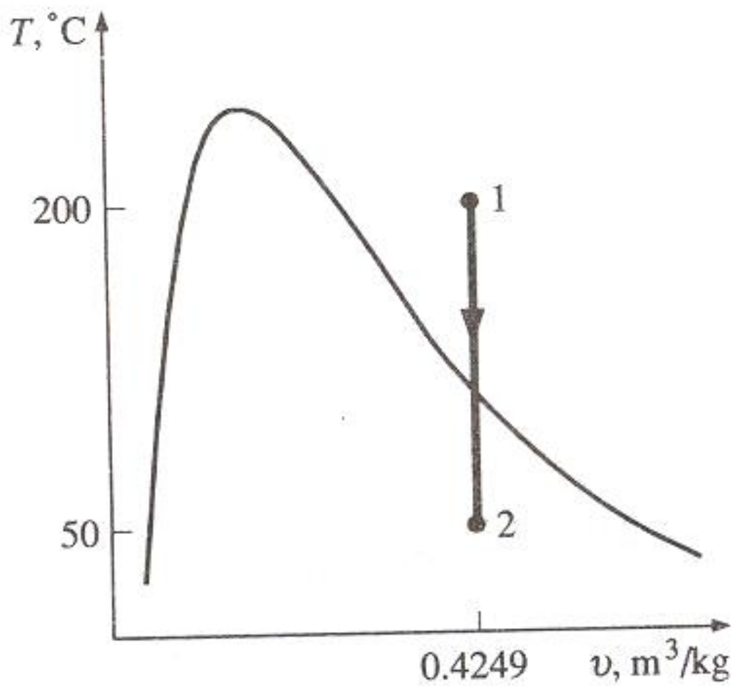
(2) $\Delta KE = 0$ (since there is no mention of velocity change).

Mass: $m_2 = m_1 = m$

Energy: $E_{in} - E_{out} = \Delta E_{system}$

$$-Q_{out} = m(u_2 - u_1)$$

Step 5: Apply relevant conservation equations and simplify them.



Step 6: Show the process on a property diagram.

Problem # 4 A piston / cylinder assembly contains $0.1m^3$ of superheated steam at 10 bar and 400^oC. If the steam is allowed to expand reversibly and adiabatically to a pressure of 3 bar, calculate the work done by the steam.

At 10 bar and 400^oC,
$v = 0.3065\ m^3/kg$
$h = 3264.4\ kJ/kg$
$s = 7.4665\ kJ/kg\ K$

At 3 bar,

$s_g = 6.9909\ kJ/kg\ K$

This is an isentropic process as initial entropy value is greater than s_g at 3 bar, the steam is superheated at the end of the process.
At 3 bar and 200^oC,
$s = 7.3119\ kJ/kg\ K$ and
at 300^oC, $s = 7.7034\ kJ/kg\ K$
therefore, the final state is having a temperature between 200^oC and 300^oC.

Equating $s_i = s_{final}$,
Find the enthalpy and specific volume by interpolation. Then calculate u_i and u_{final}.

The work done = $\Delta U = m(u_i - u_{final})$