LECTURE 5

Example 1. Consider as a system, the gas in the cylinder shown in the figure. The cylinder is fitted with a piston on which number of small weights is placed. The initial pressure is 200 kPa, and the initial volume of the gas is 0.04 m^2.

1. Let the gas in the cylinder be heated, and let the volume of the gas increase to 0.1 m^3 while the pressure remains constant. Calculate the work done by the system during this process.

$$W = \int_1^2 P\,dV$$

Since the pressure is constant, we conclude from the above equation that

$$W = P\int_1^2 dV = P(V_2 - V_1)$$

$$W = 200\,kPa * (0.1 - 0.04)m^3 = 12.0kJ$$

2. Consider the same system and initial condition, but at the same time the cylinder is being heated and the piston is rising, let the weights be removed from the piston at such a rate that, during the process the temperature of the gas remains constant.

If we assume that the ideal-gas model is valid. We note that this is a polytropic process ($PV^n = const.$) with exponent $n = 1$. For such a process,

$$W = \int_1^2 P\,dV = P_1V_1\int_1^2 \frac{dV}{V} = P_1V_1 \ln\frac{V_2}{V_1}$$

$$= 200\,kPa * 0.04\,m^3 * \ln\frac{0.10}{0.04} = 7.33\,kJ$$

3. For the same system, let the weights be removed at such a rate that the expression $PV^{1.3} = const.$ Again the final volume is 0.1 m^3. Calculate the work.

$$P = \frac{constant}{V^n} = \frac{P_1 V^n{}_1}{V^n} = \frac{P_2 V^n{}_2}{V^n}$$

$$\int_1^2 PdV = constant \int_1^2 \frac{dV}{V^n} = constant \left(\frac{V^{-n+1}}{-n+1} \right) \Bigg|_1^2$$

$$= \frac{constant}{1-n}(V_2^{1-n} - V_1^{1-n}) = \frac{P_2 V^n{}_2 V_2^{1-n} - P_1 V^n{}_1 V_1^{1-n}}{1-n}$$

$$= \frac{P_2 V_2 - P_1 V_1}{1-n}$$

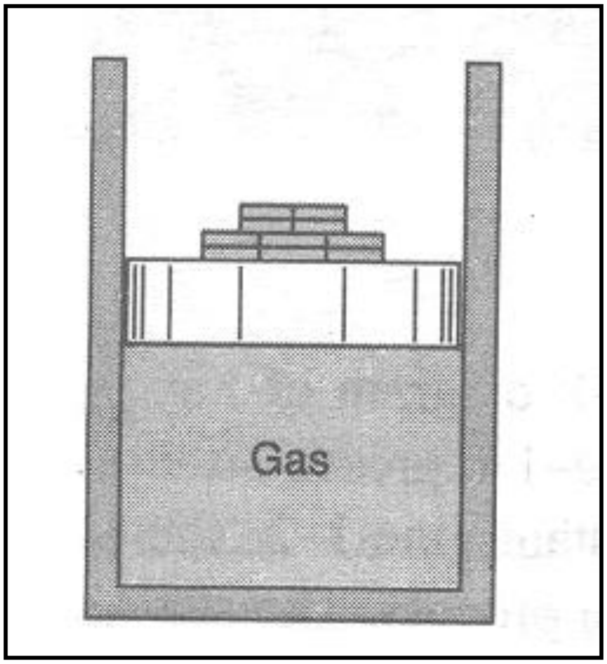
$$P_2 = 200 \left(\frac{0.04}{0.1} \right)^{1.3} = 60.77\,kPa$$

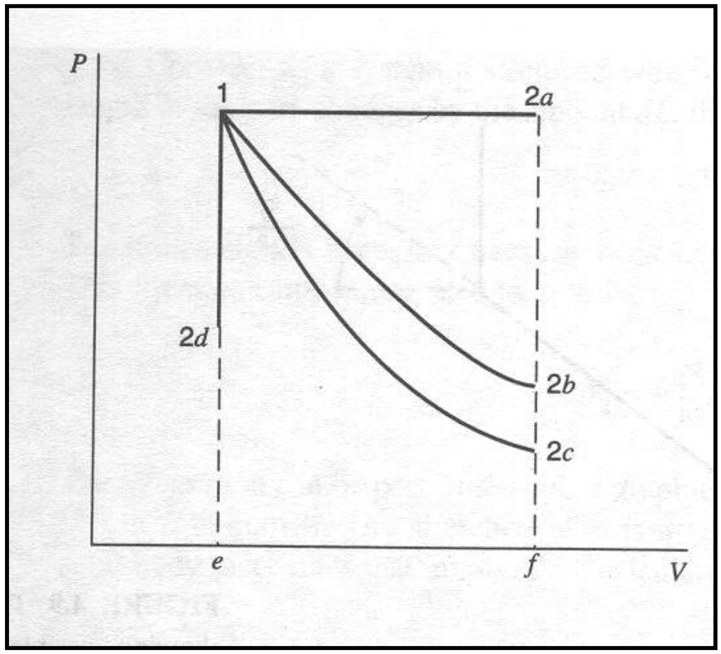
$$W = \frac{P_2 V_2 - P_1 V_1}{1-1.3} = \frac{60.77 * 0.1 - 200 * 0.04}{1-1.3}$$

$$= 6.41 kJ$$

4. Consider the system and initial state as before, but let the piston be held by a pin so that the volume remains constant. In addition, let heat transfer take place from the cylinder so that the pressure drops to 100kPa. Calculate the work.

Since dV = zero, the work is zero because there is no change in volume.





Example 2. Consider a piston cylinder arrangement as shown. The piston is loaded with a mass, m_p, the outside atmospheric pressure P_0, a linear spring and a single point force F_1. The piston is restricted in its motion by lower and upper stops trapping the gas with a pressure P.

For zero acceleration in a quasi-equilibrium process,

$$\sum F_{\uparrow} = \sum F_{\downarrow}$$

The forces when the piston is between the stops are

$$\sum F_{\uparrow} = PA \qquad \sum F_{\downarrow} = m_p g + P_0 A + k(x - x_0) + F_1$$

with the linear spring constant k

The piston position for a relaxed spring is x_0, which depends on how the spring is installed.

The force balance then gives the gas pressure by division with the area, A, as

$$P = P_0 + \left\lfloor m_p g + k(x - x_0) + F \right\rfloor / A$$

To illustrate the process in a P-V diagram, the distance x is converted to volume by division and multiplication with A:

$$P = P_0 + \frac{m_p g}{A} + \frac{F_1}{A} + \frac{k}{A^2}(V - V_0) = C_1 + C_2 V$$

This relation gives the pressure as a linear function of the volume with slope of $C_2 = k/A^2$

The work in a quasi-equilibrium process, is

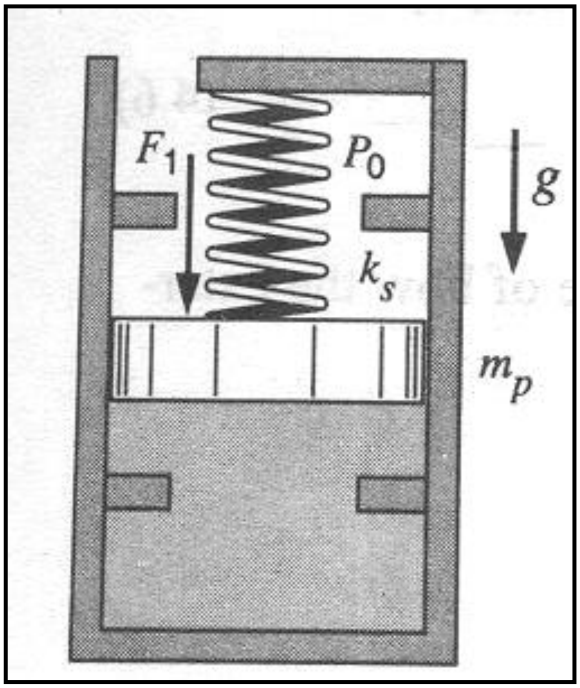
$$W = \int_1^2 P\,dV = Area\ under\ the\ curve$$

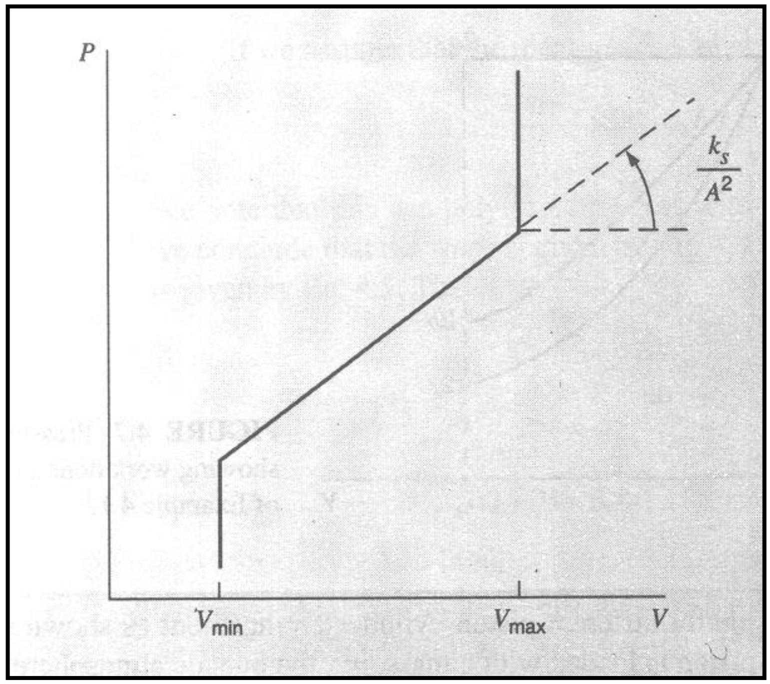
$$= \frac{1}{2}\left(P_1^{`} + P_2^{`}\right)\left(V_2 - V_1\right)$$

With $P^{`}_1 = P_1$ and $P'_2 = P_2$ subject to the constraint that

$$P_{\min} \le P_1^{`}, \quad P_2^{`} \le P_{\max}$$

An part of the process with a pressure smaller that P_{min} or larger than P_{max} does not involve work.





Example 3. Balloon initially collapsed and flat. Slowly filled with helium from a cylinder

Dia = 5 m.
Ambient pressure = 100 kPa
Temperature const. = 300 K

System: Helium contained in cylinder and balloon.

Filling process **quasi-static**

Work done by Helium , W

$$= \int_1^2 P\,dV = P(V_2 - V_1)$$

$$= PV_2$$

$$= 100 * 10^3 * \frac{4\pi}{3}(5/2)^3 = 6544.98 kJ$$