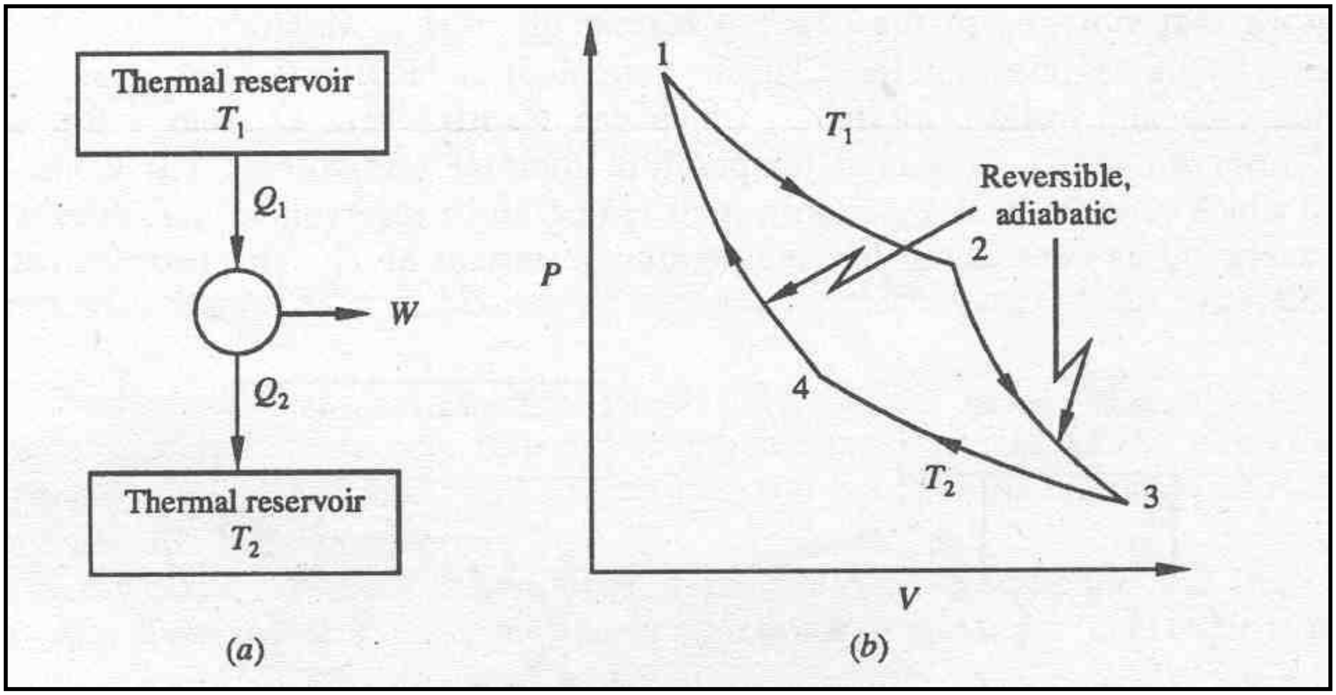
CARNOT CYCLE



The Carnot cycle uses only two thermal reservoirs – one at high temperature T_1 and the other at two temperature T_2.

If the process undergone by the working fluid during the cycle is to be reversible, the heat transfer must take place with no temperature difference, i.e. it should be isothermal.

The Carnot cycle consists of a reversible isothermal expansion from state 1 to 2, reversible adiabatic expansion from state 2 to 3, a reversible isothermal compression from state 3 to 4 followed by a reversible adiabatic compression to state 1.

The thermal efficiency, η is given by

η = Net work done / Energy absorbed as heat

During processes 2-3 and 4-1, there is no heat interaction as they are adiabatic.

$$Q_{1-2} = \int_1^2 Pdv = \int_1^2 RT_1 \frac{dv}{v} = RT_1 \ln(v_2 / v_1)$$

Similarly for the process 3-4,

$$Q_{3-4} = \int_3^4 Pdv = \int_3^4 RT_2 \frac{dv}{v} = RT_2 \ln(v_4 / v_3)$$

Net heat interaction = Net work done

$= RT_1 \ln(v_2/v_1) + RT_2 \ln(v_4/v_3)$

$= RT_1 \ln(v_2/v_1) - RT_2 \ln(v_3/v_4)$

The processes 2-3 and 4-1 are reversible, adiabatic and hence

$T_1 v_2^{\gamma-1} = T_2 v_3^{\gamma-1}$

Or, $v_2/v_3 = (T_2/T_1)^{1/(\gamma-1)}$

And $T_2 v_4^{\gamma-1} = T_1 v_1^{\gamma-1}$

Or, $v_1/v_4 = (T_2/T_1)^{1/(\gamma-1)}$

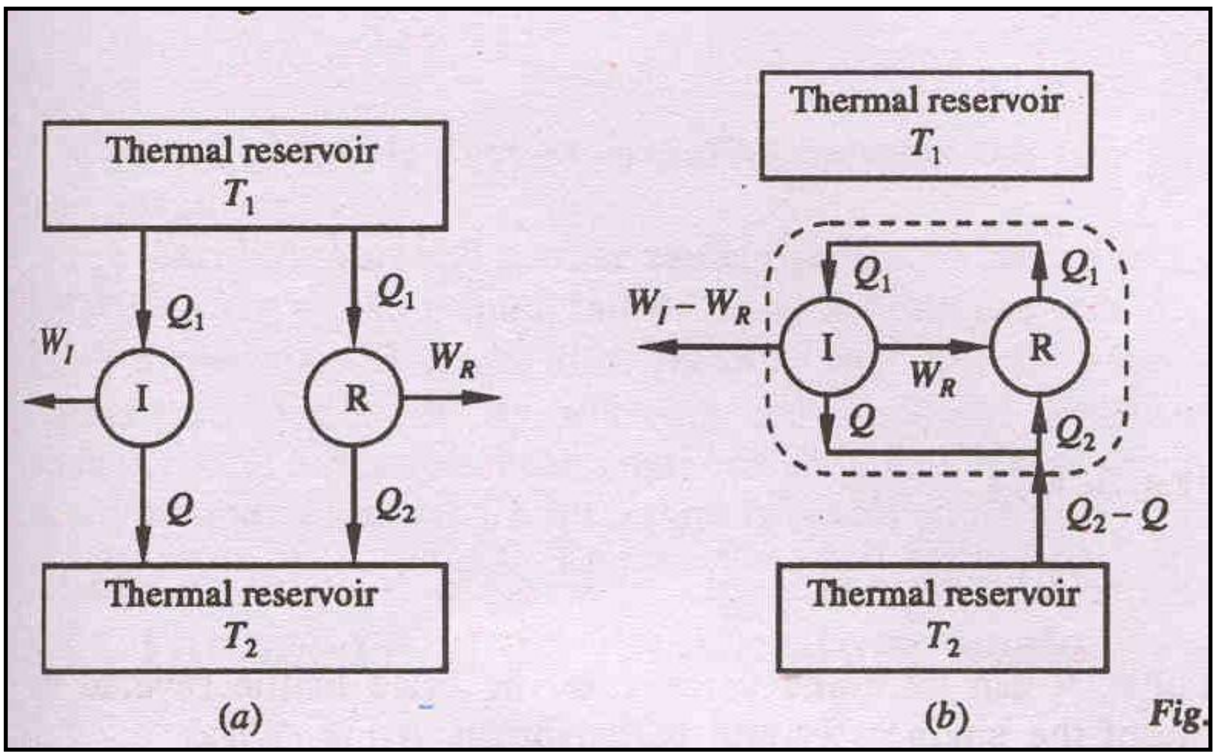
$v_2/v_3 = v_1/v_4$ or $v_2/v_1 = v_3/v_4$

$\eta = \{RT_1 \ln(v_2/v_1) - RT_2 \ln(v_3/v_4)\} / RT_1 \ln(v_2/v_1)$

$\eta = (T_1 – T_2)/T_1$
$= 1- T_2/T_1$

The Carnot Principles

1. The efficiency of an irreversible heat engine is always less than the efficiency of a reversible one operating between same two thermal reservoirs.
2. The efficiencies of all reversible heat engines operating between the same two thermal reservoirs are the same.



Lets us assume it is possible for an engine I to have an efficiency greater than the efficiency of a reversible heat engine R.

$$\eta_I > \eta_R$$

Let both the engines absorb same quantity of energy Q_1. Let Q and Q_2 represent the energy rejected as heat by the engines R, and I respectively.

$W_I = Q_1 - Q$
$W_R = Q_1 - Q_2$

$\eta_I = W_I / Q_1 = (Q_1 - Q)/Q_1 = 1 - Q/Q_1$

$\eta_R = W_R/Q_1 = (Q_1 - Q_2)/Q_1 = 1 - Q_2/Q_1$

Since $\eta_I > \eta_R$,

$1 - Q/Q_1 > 1 - Q_2/Q_1$

or, $Q < Q_2$

Therefore, W_I (= Q_1-Q) > W_R (=$Q_1 - Q_2$)

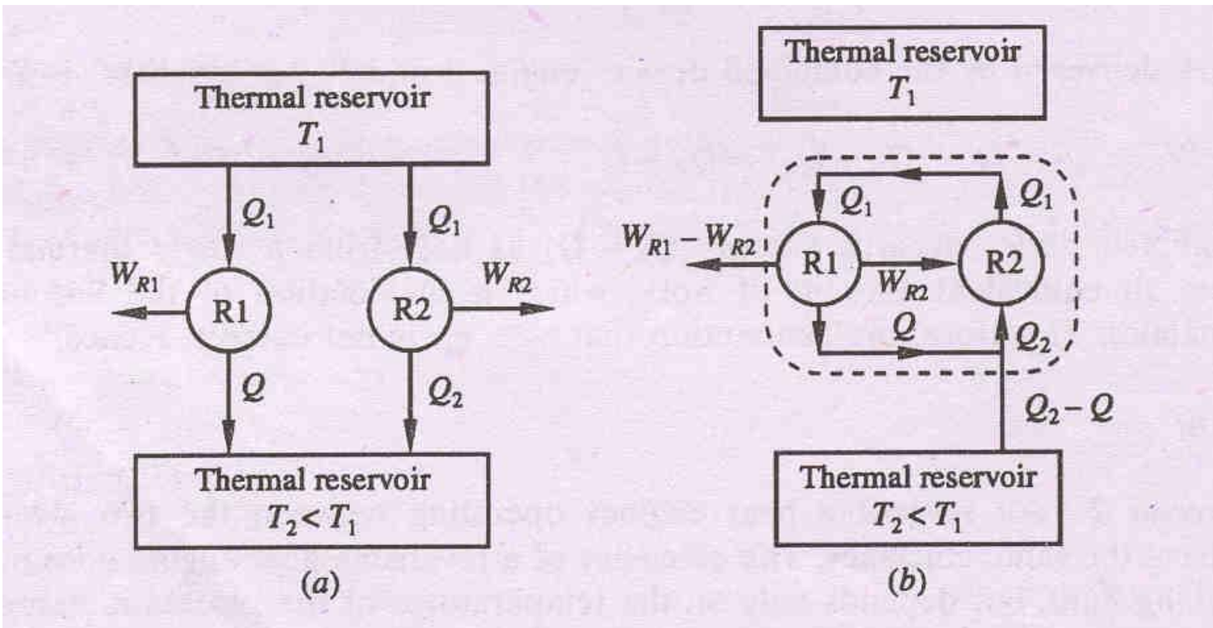
Since the engine R is reversible, it can be made to execute in the reverse order. Then, it will absorb energy Q_2 from the reservoir at T_2 and reject energy Q_1 to the reservoir at T_1 when work W_R is done on it.

If now engines I and R are combined, the net work delivered by the combined device is given by

$$W_I - W_R = Q_1 - Q - (Q_1 - Q_2) = Q_2 - Q$$

The combined device absorbs energy ($Q_2 - Q$) as heat from a single thermal reservoir and delivers an equivalent amount of work, which violates the second law of thermodynamics.

Hence, $\eta_R \geq \eta_I$



Carnot principle 2

Consider two reversible heat engines R_1 and R_2 , operating between the two given thermal reservoirs at temperatures T_1 and T_2.

Let $\eta_{R1} > \eta_{R2}$

Q_1= energy absorbed as heat from the reservoir at T1 by the engines R_1 and R_2, separately.

Q = energy rejected by reversible engine R_1 to the reservoir at T_2

Q_2 = energy rejected by reversible engine R_2 to the reservoir at T_2.

$W_{R1} = Q_1 - Q$ = work done by a reversible engine R_1 .

$W_{R2} = Q_1 - Q_2$ = work done by a reversible engine R_2

According to assumption,

$\eta_{R1} > \eta_{R2}$

Or, $1 - Q/Q_1 > 1 - Q_2/Q_1$

$Q_1 - Q > Q_1 - Q_2$ or $W_{R1} > W_{R2}$

$W_{R1} - W_{R2} = (Q_1 - Q) - (Q_1 - Q_2) = Q_2 - Q$

Since the engine R_2 is reversible, it can be made to execute the cycle in the reverse by supplying W_{R2}.

Since $W_{R1} > W_{R2}$ the reversible engine R_2 can be run as a heat pump by utilizing part of the work delivered by R_1.
For the combined device,

$W_{R1} – W_{R2} = Q_2 – Q$, by absorbing energy $Q_2 – Q$ from a single thermal reservoir which violates the second law of thermodynamics.

Hence $\eta_{R1} > \eta_{R2}$ is incorrect.

By similar arguments, if we assume that $\eta_{R2} > \eta_{R1}$ then,
$\eta_{R1} \geq \eta_{R2}$

Therefore, based on these two equations,
$\eta_{R1} = \eta_{R2}$

The efficiency of a reversible heat engine is also independent of the working fluid and depends only on the temperatures of the reservoirs between which it operates.