## Assignment (Unit-I)

1. Find the formula for the $n^{\text {th }}$ term of the sequence and what can you say about its convergence.
a) $1,-\frac{1}{4}, \frac{1}{9},-\frac{1}{16}, \frac{1}{25}, \ldots$
b) $\frac{1}{9}, \frac{2}{12}, \frac{2^{2}}{15} \cdot \frac{2^{3}}{18}, \ldots$
c) $\frac{100}{2},-\frac{100^{2}}{6}, \frac{100^{3}}{24},-\frac{100^{4}}{120}, \ldots$
2. Determine whether the sequence $\left\{a_{n}\right\}$ is monotonic, bounded and convergent, where
(a) $a_{n}=\frac{3 n+1}{n+1}$,
(b) $a_{n}=\frac{(2 n+3)!}{(n+1)!}$
(C) $a_{n}=\frac{n!}{n^{n}}$

3 Discuss the convergence of the sequence $\left\{a_{n}\right\}$ where
a) $a_{n}=\frac{n+1}{n}$
b) $a_{n}=\frac{n}{n^{2}+1}$
c) $a_{n}=1+\frac{1}{3}+\frac{1}{3^{2}}+\ldots \frac{1}{3^{n}}$

4 Check the following positive term series for convergence and divergence
a) $\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{n^{2}}$
b) $\sum_{n=1}^{\infty} \frac{n+2^{n}}{n^{2} \cdot 2^{n}}$
c) $\sum_{n=1}^{\infty} \frac{1 \cdot 3 \ldots(2 n-1)}{4^{n} 2^{n} n!}$
d) $\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \ldots(2 n-1)}{2 \cdot 4 \cdot 6 \ldots 2 n\left(3^{n}+1\right)}$

5 State Leibniz's test of convergence for alternating series of real numbers.

6 Using integral test show that $\sum_{n=1}^{\infty} \frac{1}{n^{p}}$ is convergent when $p>1$ and diverges for $p \leq 1$.
7 Define absolute and conditional convergence of a series with arbitrary terms.

8 Check the following series for absolutely or conditionally convergence,
a) $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{1}{n^{2}}$
b) $\sum_{n=1}^{\infty} \frac{\sin n \alpha}{n^{2}}$
c) $\sum_{n=1}^{\infty} \frac{(-1)^{n}(n+2)}{2^{n}+5}$
d) $1-\frac{x^{2}}{2!}+\frac{x^{4}}{4!}-\frac{x^{6}}{6!}+\ldots$
e) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\log (1+n)}$

