BAYE'S THEOREM

Statement → If $A_1, A_2, A_3 \dots A_n$ are n mutually exclusive and exhaustive events such that $P(A_i) \neq 0$ $(i = 1, 2, 3 \dots n)$ of a random experiment then for any arbitrary event B of the sample space of the above experiment with $P(B) > 0$ then

$$P(A_i/B) = \frac{P(A_i).P(B/A_i)}{\sum_{i=1}^{n} P(A_i)\, P(B/A_i)} \quad (1 \le i \le n)$$

Proof → From the Multiplication theorem of Compound Probability

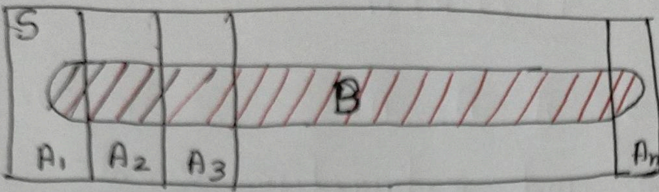
$$\left.\begin{aligned} P(A_i \cap B) &= P(B)\, P(A_i/B) \\ \text{and also } P(A_i \cap B) &= P(A_i)\, P(B/A_i) \end{aligned}\right\} \rightarrow (1)$$

Equating the two values of $P(A_i \cap B)$ we get

$$P(B).P(A_i/B) = P(A_i).P(B/A_i) \rightarrow (2)$$

$$P(A_i/B) = \frac{P(A_i)\, P(B/A_i)}{P(B)}$$

Since the events $A_1, A_2, A_3 \dots A_n$ are mutual exclusive & exhaustive, as shown in figure so 'B' can occur with only one of these events
so that we have from figure



here S be the sample space of random Experiment and Events A_1, A_2, A_3 — — A_n being exhaustive

We have

here $B \subset S$

$$S = A_1 \cup A_2 \cup A_3 - \text{———} \cup A_n$$

$$\& \; B = B \cap S$$

$$B = (A_1 \cap B) \cup (A_2 \cap B) \cup (A_3 \cap B) - \text{———} \cup (A_n \cap B)$$

here the Events $A_1 \cap B$, $A_2 \cap B$, ——— $A_n \cap B$ are also mutually Exclusive events.

Therefore, using the addition Theorem of Probability

$$P(B) = P(A_1 \cap B) + P(A_2 \cap B) + \text{——} + P(A_n \cap B)$$

$$= \sum_{i=1}^{n} P(A_i \cap B)$$

use Multiplication Theorem $\boxed{\text{using } ①}$

$$P(B) = \sum_{i=1}^{n} P(A_i).P\left(\frac{B}{A_i}\right) \rightarrow ③$$

using ③ in Eqn ②

$$P(A_i/B) = \frac{P(A_i).P(B/A_i)}{\sum_{i=1}^{n} P(A_i)P(B/A_i)}$$

Note → $P(A_i)$ is the probability of occurrence of A_i. The exp is performed & we are told that the event B has occurred. With information, the probability $P(A_i)$ is changed to $P(A_i/B)$. Baye's th enables us to Evaluate $P(A_i/B)$ if all the $P(A_i)$ and the Conditional Probability P