Mathematical Expectation

$$E(X+Y) = E(X) + E(Y)$$

The expected value of the sum of two variables is equal to the sum of their expected value.

Similarly → $E(X+Y+Z) = E(X) + E(Y) + E(Z)$

Expected Value of a Constant

$$E(C) = C$$

Theorem → First Moment about mean is Zero

$$E(X) = \sum xp = \sum_{i=1}^{n} p_i x_i$$

x	x_1	x_2	———	x_n
p	p_1	p_2	———	p_n

and $\mu_1 = E[X - E(X)]$ Remember

$= E[X + \{-E(X)\}]$ $\because E(X+Y) = E(X) + E(Y)$ and $E(C) = C$

$= E(X) - [E(X)]$ $\because E(-1) = -1$

$= E(X) - E(X)$

$= 0$

Variance $\rightarrow$ $E(x) = \sum_{i=1}^{n} p_i x_i$

$$\mu_2 = \text{Var } X = E[X - E(X)]^2$$

$$= E[X^2 - 2XE(x) + \{E(x)\}^2]$$

$\because X = E(X)$ $$= E(X^2) - 2E(X)E(X) + [E(X)]^2$$

$$= E(X^2) - 2[E(X)]^2 + [E(X)]^2$$

$$= E(X^2) - [E(X)]^2$$

$$= E(X^2) - [EX]^2$$

Example → $p(x) = \frac{1}{2}$, $x = 2, 4$

$= 0$ otherwise

Find

(i) $E(X)$ (ii) $E\left(\frac{1}{X}\right)$ (iii) $E(X^2)$

Solution → when $x = 2, 4$

$$E(X) = \sum_{x=1}^{4} x\,p(x) = 1\,p(1) + 2\,p(2) + 3\,p(3) + 4\,p(4)$$

$$= 0 + 2 \cdot \frac{1}{2} + 0 + 4\left(\frac{1}{2}\right)$$

$$= 1 + 2 = 3$$

(ii) $$E\left(\frac{1}{X}\right) = \sum_{x=1}^{4} \frac{1}{x}\,p(x) = \frac{1}{1}P(1) + \frac{1}{2}P(2) + \frac{1}{3}P(3) + \frac{1}{4}P(4)$$

$$= 0 + \frac{1}{2} \cdot \frac{1}{2} + 0 + \frac{1}{4}\left(\frac{1}{2}\right)$$

$$= \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$$

(iii) $$E(X^2) = \sum x^2\,p(x) = 1^2 P(1) + 2^2 P(2) + 3^2 P(3) + 4^2 P(4)$$

$$0 + 4P(2) + 0 + 4^2 P(4)$$

$$0 + 4\left(\frac{1}{2}\right) + 0 + 16 \cdot \frac{1}{2}$$

$$= 2 + 8$$

$$= 10$$