

## THE TIME VALUE OF MONEY

1. Calculate the value 10 years hence of a deposit of Rs. 20,000 made today if the interest rate is (a) 4 percent, (b) 6 percent, (c) 8 percent, and (d) 9 percent.

**Solution:**

Value 10 years hence of a deposit of Rs. 20,000 at various interest rates is as follows:

$$\begin{aligned} r &= 4 \% & FV_5 &= 20,000 \times FVIF (4\%, 10 \text{ years}) \\ & & &= 20,000 \times 1.480 \\ & & &= \text{Rs.}29,600 \end{aligned}$$

$$\begin{aligned} r &= 6 \% & FV_5 &= 20,000 \times FVIF (6 \%, 10 \text{ years}) \\ & & &= 20,000 \times 1.791 \\ & & &= \text{Rs.}35,820 \end{aligned}$$

$$\begin{aligned} r &= 8 \% & FV_5 &= 20,000 \times FVIF (8 \%, 10 \text{ years}) \\ & & &= 20,000 \times 2.159 \\ & & &= \text{Rs.}43,180 \end{aligned}$$

$$\begin{aligned} r &= 9 \% & FV_5 &= 20,000 \times FVIF (9 \%, 10 \text{ years}) \\ & & &= 20,000 \times 2.367 \\ & & &= \text{Rs.} 47,340 \end{aligned}$$

2. Calculate the value 3 years hence of a deposit of Rs.5,800 made today if the interest rate is (a) 12 percent, (b)14 percent, (c) 15 percent, and (d) 16 percent.

**Solution:**

Value 3 years hence of a deposit of Rs. 5,800 at various interest rates is as follows:

$$\begin{aligned} r &= 12 \% & FV_5 &= 5,800 \times FVIF (12\%, 3 \text{ years}) \\ & & &= 5,800 \times 1.405 \\ & & &= \text{Rs.}8,149 \end{aligned}$$

$$\begin{aligned} r &= 14 \% & FV_5 &= 5,800 \times FVIF (14\%, 3 \text{ years}) \\ & & &= 5,800 \times 1.482 \\ & & &= \text{Rs.}8,596 \end{aligned}$$

$$\begin{aligned} r &= 15 \% & FV_5 &= 5,800 \times FVIF (15\%, 3 \text{ years}) \\ & & &= 5,800 \times 1.521 \\ & & &= \text{Rs.}8,822 \end{aligned}$$

$$\begin{aligned} r &= 16 \% & FV_5 &= 5,800 \times FVIF (16\%, 3 \text{ years}) \\ & & &= 5,800 \times 1.561 \\ & & &= \text{Rs.} 9,054 \end{aligned}$$

3. If you deposit Rs.2,000 today at 6 percent rate of interest in how many years (roughly) will this amount grow to Rs.32,000 ? Work this problem using the *rule of 72*—do not use tables.

**Solution:**

$$\text{Rs.}32,000 / \text{Rs. } 2,000 = 16 = 2^4$$

According to the Rule of 72 at 6 percent interest rate doubling takes place approximately in  $72 / 6 = 12$  years

So Rs.2,000 will grow to Rs.32,000 in approximately  $4 \times 12$  years = 48 years

4. If you deposit Rs.3,000 today at 8 percent rate of interest in how many years (roughly) will this amount grow to Rs.1,92,000 ? Work this problem using the *rule of 72*—do not use tables.

**Solution:**

$$\text{Rs.}192,000 / \text{Rs. } 3,000 = 64 = 2^6$$

According to the Rule of 72 at 8 percent interest rate doubling takes place approximately in  $72 / 8 = 9$  years

So Rs.3000 will grow to Rs.192,000 in approximately  $6 \times 9$  years = 54 years

5. A finance company offers to give Rs.20,000 after 14 years in return for Rs.5,000 deposited today. Using the *rule of 69*, figure out the approximate interest rate offered.

**Solution:**

In 14 years Rs.5,000 grows to Rs.20,000 or 4 times. This is  $2^2$  times the initial deposit. Hence doubling takes place in  $14 / 2 = 7$  years.

According to the Rule of 69, the doubling period is  $0.35 + 69 / \text{Interest rate}$   
We therefore have

$$\begin{aligned} 0.35 + 69 / \text{Interest rate} &= 7 \\ \text{Interest rate} &= 69 / (7 - 0.35) = 10.38 \% \end{aligned}$$

6. Someone offers to give Rs.80,000 to you after 18 years in return for Rs.10,000 deposited today. Using the *rule of 69*, figure out the approximate interest rate offered.

**Solution:**

In 18 years Rs.10,000 grows to Rs.80,000 or 8 times. This is  $2^3$  times the initial deposit. Hence doubling takes place in  $18 / 3 = 6$  years.

According to the Rule of 69, the doubling period is  $0.35 + 69 / \text{Interest rate}$ . We therefore have

$$\begin{aligned} 0.35 + 69 / \text{Interest rate} &= 6 \\ \text{Interest rate} &= 69 / (6 - 0.35) = 12.21 \% \end{aligned}$$

7. You can save Rs.5,000 a year for 3 years, and Rs.7,000 a year for 7 years thereafter. What will these savings cumulate to at the end of 10 years, if the rate of interest is 8 percent?

**Solution:**

Saving Rs.5000 a year for 3 years and Rs.6000 a year for 7 years thereafter is equivalent to saving Rs.5000 a year for 10 years and Rs.2000 a year for the years 4 through 10.

Hence the savings will cumulate to:

$$\begin{aligned}
 & 5000 \times \text{FVIFA} (8\%, 10 \text{ years}) + 2000 \times \text{FVIFA} (8\%, 7 \text{ years}) \\
 = & 5000 \times 14.487 + 2000 \times 8.923 \\
 = & \text{Rs.90281}
 \end{aligned}$$

8. Krishna saves Rs. 24,000 a year for 5 years, and Rs.30,000 a year for 15 years thereafter. If the rate of interest is 9 percent compounded annually, what will be the value of his savings at the end of 20 years?

**Solution:**

Saving Rs.24,000 a year for 5 years and Rs.30,000 a year for 15 years thereafter is equivalent to saving Rs.24,000 a year for 20 years and Rs.6,000 a year for the years 6 through 20.

Hence the savings will cumulate to:

$$\begin{aligned}
 & 24,000 \times \text{FVIFA} (9\%, 20 \text{ years}) + 6,000 \times \text{FVIFA} (9\%, 15 \text{ years}) \\
 = & 24,000 \times 51.160 + 6,000 \times 29.361 \\
 = & \text{Rs. 1,404,006}
 \end{aligned}$$

9. You plan to go abroad for higher studies after working for the next five years and understand that an amount of Rs.2, 000,000 will be needed for this purpose at that time. You have decided to accumulate this amount by investing a fixed amount at the end of each year in a safe scheme offering a rate of interest at 10 percent. What amount should you invest every year to achieve the target amount?

**Solution:**

Let A be the annual savings.

$$\begin{aligned}
 A \times \text{FVIFA} (10\%, 5 \text{ years}) & = 2,000,000 \\
 A \times 6.105 & = 2,000,000
 \end{aligned}$$

$$\text{So, } A = 2,000,000 / 6.105 = \text{Rs. 327,600}$$

10. How much should Vijay save each year, if he wishes to purchase a flat expected to cost Rs.80 lacs after 8 years, if the investment option available to him offers a rate of interest at 9 percent? Assume that the investment is to be made in equal amounts at the end of each year.

**Solution:**

Let A be the annual savings.

$$A \times \text{FVIFA} (9\%, 8 \text{ years}) = 80,00,000$$

$$A \times 11.028 = 80,00,000$$

$$\text{So, } A = 80,00,000 / 11.028 = \text{Rs. } 7,25,426$$

11. A finance company advertises that it will pay a lump sum of Rs.100,000 at the end of 5 years to investors who deposit annually Rs.12,000. What interest rate is implicit in this offer?

**Solution:**

$$12,000 \times \text{FVIFA} (r, 5 \text{ years}) = 100,000$$

$$\text{FVIFA} (r, 5 \text{ years}) = 100,000 / 12,000$$

$$= 8.333$$

From the tables we find that

$$\text{FVIFA} (24\%, 5 \text{ years}) = 8.048$$

$$\text{FVIFA} (28\%, 5 \text{ years}) = 8.700$$

Using linear interpolation in the interval, we get:

$$r = 24\% + \frac{(8.333 - 8.048)}{(8.700 - 8.048)} \times 4\% = 25.75\%$$

12. Someone promises to give you Rs.5,000,000 after 6 years in exchange for Rs.2,000,000 today. What interest rate is implicit in this offer?

**Solution:**

$$2,000,000 \times \text{FVIF} (r, 6 \text{ years}) = 5,000,000$$

$$\text{FVIF} (r, 6 \text{ years}) = 5,000,000 / 2,000,000 = 2.5$$

From the tables we find that

$$\text{FVIF} (16\%, 6 \text{ years}) = 2.436$$

$$\text{FVIF} (17\%, 6 \text{ years}) = 2.565$$

Using linear interpolation in the interval, we get:

$$r = 16\% + \frac{(2.5 - 2.436) \times 1\%}{(2.565 - 2.436)} = 16.5\%$$

13. At the time of his retirement, Rahul is given a choice between two alternatives: (a) an annual pension of Rs. 120,000 as long as he lives, and (b) a lump sum amount of Rs.1,000,000. If Rahul expects to live for 20 years and the interest rate is expected to be 10 percent throughout, which option appears more attractive?

**Solution:**

The present value of an annual pension of Rs.120,000 for 20 years when  $r = 10\%$  is:

$$120,000 \times \text{PVIFA} (10\%, 20 \text{ years})$$

$$= 120,000 \times 8.514$$

$$= \text{Rs.}1,021,680$$

The alternative is to receive a lumpsum of Rs 1,000,000

Rahul will be better off with the annual pension amount of Rs.120,000.

14. A leading bank has chosen you as the winner of its quiz competition and asked you to choose from one of the following alternatives for the prize: (a) Rs. 60,000 in cash immediately or (b) an annual payment of Rs. 10,000 for the next 10 years. If the interest rate you can look forward to for a safe investment is 9 percent, which option would you choose

**Solution:**

The present value of an annual payment of Rs.10,000 for 10 years when  $r = 9\%$  is:

$$10,000 \times \text{PVIFA} (9\%, 10 \text{ years})$$

$$= 10,000 \times 6.418$$

$$= \text{Rs.} 64,180$$

The annual payment option would be the better alternative

15. What is the present value of an income stream which provides Rs.30,000 at the end of year one, Rs.50,000 at the end of year three, and Rs.100,000 during each of the years 4 through 10, if the discount rate is 9 percent ?

**Solution:**

The present value of the income stream is:

$$30,000 \times \text{PVIF} (9\%, 1 \text{ year}) + 50,000 \times \text{PVIF} (9\%, 3 \text{ years})$$

$$+ 100,000 \times \text{PVIFA} (9\%, 7 \text{ years}) \times \text{PVIF} (9\%, 3 \text{ years})$$

$$= 30,000 \times 0.917 + 50,000 \times 0.772 + 100,000 \times 5.033 \times 0.0.772 = \text{Rs.}454,658.$$

16. What is the present value of an income stream which provides Rs.25,000 at the end of year one, Rs.30,000 at the end of years two and three, and Rs.40,000 during each of the years 4 through 8 if the discount rate is 15 percent ?

**Solution:**

The present value of the income stream is:

$$\begin{aligned} & 25,000 \times \text{PVIF} (15\%, 1 \text{ year}) + 30,000 \times \text{PVIF} (15\%, 2 \text{ years}) \\ & + 30,000 \times \text{PVIF} (15\%, 3 \text{ years}) \\ & + 40,000 \times \text{PVIFA} (15\%, 5 \text{ years}) \times \text{PVIF} (15\%, 3 \text{ years}) \\ & = 25,000 \times 0.870 + 30,000 \times 0.756 + 30,000 \times 0.658 \\ & + 40,000 \times 3.352 \times 0.658 = \text{Rs.}152,395. \end{aligned}$$

17. What is the present value of an income stream which provides Rs.1,000 a year for the first three years and Rs.5,000 a year forever thereafter, if the discount rate is 12 percent?

**Solution:**

The present value of the income stream is:

$$\begin{aligned} & 1,000 \times \text{PVIFA} (12\%, 3 \text{ years}) + (5,000/ 0.12) \times \text{PVIF} (12\%, 3 \text{ years}) \\ & = 1,000 \times 2.402 + (5000/0.12) \times 0.712 \\ & = \text{Rs.}32,069 \end{aligned}$$

18. What is the present value of an income stream which provides Rs.20,000 a year for the first 10 years and Rs.30,000 a year forever thereafter, if the discount rate is 14 percent ?

**Solution:**

The present value of the income stream is:

$$\begin{aligned} & 20,000 \times \text{PVIFA} (14\%, 10 \text{ years}) + (30,000/ 0.14) \times \text{PVIF} (14\%, 10 \text{ years}) \\ & = 20,000 \times 5.216 + (30,000/0.14) \times 0.270 \\ & = \text{Rs.}162,177 \end{aligned}$$

19. Mr. Ganapathi will retire from service in five years .How much should he deposit now to earn an annual income of Rs.240,000 forever beginning from the end of 6 years from now ? The deposit earns 12 percent per year.

**Solution:**

To earn an annual income of Rs.240,000 forever , beginning from the end of 6 years from now, if the deposit earns 12% per year a sum of

Rs. 240,000 / 0.12 = Rs.2,000,000 is required at the end of 5 years. The amount that must be deposited to get this sum is:

$$\begin{aligned} \text{Rs.2,000,000 PVIF (12\%, 5 years)} &= \text{Rs.2,000,000} \times 0.567 \\ &= \text{Rs. 1,134,000} \end{aligned}$$

20. Suppose someone offers you the following financial contract. If you deposit Rs.100,000 with him he promises to pay Rs.50,000 annually for 3 years. What interest rate would you earn on this deposit?

**Solution:**

$$\begin{aligned} \text{Rs.100,000} &= \text{Rs.50,000} \times \text{PVIFA (r, 3 years)} \\ \text{PVIFA (r,3 years)} &= 2.00 \end{aligned}$$

From the tables we find that:

$$\begin{aligned} \text{PVIFA (20 \% , 3 years)} &= 2.106 \\ \text{PVIFA (24 \% , 3 years)} &= 1.981 \end{aligned}$$

Using linear interpolation we get:

$$\begin{aligned} r &= 20 \% + \left[ \frac{2.106 - 2.00}{2.106 - 1.981} \right] \times 4\% \\ &= 23.39 \% \end{aligned}$$

21. If you invest Rs.600,000 with a company they offer to pay you Rs.100,000 annually for 10 years. What interest rate would you earn on this investment?

**Solution:**

$$\begin{aligned} \text{Rs.600,000} &= \text{Rs.100,000} \times \text{PVIFA (r, 10 years)} \\ \text{PVIFA (r, 10 years)} &= 6.00 \end{aligned}$$

From the tables we find that:

$$\begin{aligned} \text{PVIFA (10 \% , 10 years)} &= 6.145 \\ \text{PVIFA (11 \% , 10 years)} &= 5.889 \end{aligned}$$

Using linear interpolation we get:

$$r = 10 \% + \left[ \frac{6.145 - 6.00}{6.145 - 5.889} \right] \times 1\%$$
$$= 10.57 \%$$



22. What is the present value of the following cash flow streams?

End of year	Stream X	Stream Y	Stream Z
1	500	750	600
2	550	700	600
3	600	650	600
4	650	600	600
5	700	550	600
6	750	500	600

-----  
The discount rate is 18 percent.

**Solution:**

$$\begin{aligned}
 \text{PV( Stream X)} &= 500 \text{ PV( 18\%, 1yr)} + 550 \text{ PV( 18\%, 2yrs)} + 600 \text{ PV( 18\%, 3yrs)} + 650 \text{ PV( 18\%, 4yrs)} \\
 &+ 700 \text{ PV( 18\%, 5yrs)} + 750 \text{ PV( 18\%, 6yrs)} \\
 &= 500 \times 0.847 + 550 \times 0.718 + 600 \times 0.609 + 650 \times 0.516 + 700 \times 0.437 + 750 \times 0.370 \\
 &= 2102.6
 \end{aligned}$$

$$\begin{aligned}
 \text{PV( Stream X)} &= 750 \text{ PV( 18\%, 1yr)} + 700 \text{ PV( 18\%, 2yrs)} + 650 \text{ PV( 18\%, 3yrs)} + 600 \text{ PV( 18\%, 4yrs)} \\
 &+ 550 \text{ PV( 18\%, 5yrs)} + 500 \text{ PV( 18\%, 6yrs)} \\
 &= 750 \times 0.847 + 700 \times 0.718 + 650 \times 0.609 + 600 \times 0.516 + 550 \times 0.437 + 500 \times 0.370 \\
 &= 2268.65
 \end{aligned}$$

$$\text{PV( Stream X)} = 600 \text{ PVIFA (18\%, 6yrs)} = 600 \times 3.498 = 2098.8$$

23. Suppose you deposit Rs.200,000 with an investment company which pays 12 percent interest with compounding done once in every two months, how much will this deposit grow to in 10 years?

**Solution:**

$$\begin{aligned}
 \text{FV}_{10} &= \text{Rs.}200,000 [1 + (0.12 / 6)]^{10 \times 6} \\
 &= \text{Rs.}200,000 (1.02)^{60} \\
 &= \text{Rs.}200,000 \times 3.281 \\
 &= \text{Rs.}656,200
 \end{aligned}$$

24. A bank pays interest at 5 percent on US dollar deposits, compounded once in every six months. What will be the maturity value of a deposit of US dollars 15,000 for three years?

**Solution:**

$$\begin{aligned}
 \text{Maturity value} &= \text{USD } 15,000 [1 + (0.05 / 2)]^{3 \times 2} \\
 &= 15,000 (1.025)^6 \\
 &= 15,000 \times 1.1597 \\
 &= 17,395.50
 \end{aligned}$$

25. What is the difference between the effective rate of interest and stated rate of interest in the following cases:

*Case A:* Stated rate of interest is 8 percent and the frequency of compounding is six times a year.

*Case B:* Stated rate of interest is 10 percent and the frequency of compounding is four times a year.

*Case C:* Stated rate of interest is 12 percent and the frequency of compounding is twelve times a year.

**Solution:**

	A	B	C
Stated rate (%)	8	10	12
Frequency of compounding	6 times	4 times	12 times
Effective rate (%)	$(1 + 0.08/6)^6 - 1$ = 8.27	$(1+0.10/4)^4 - 1$ = 10.38	$(1 + 0.12/12)^{12} - 1$ = 12.68
Difference between the effective rate and stated rate (%)	0.27	0.38	0.68

26. You have a choice between Rs.200,000 now and Rs.600,000 after 8 years. Which would you choose? What does your preference indicate?

**Solution:**

The interest rate implicit in the offer of Rs.600,000 after 8 years in lieu of Rs.200,000 now is:

$$\text{Rs.200,000} \times \text{FVIF}(r, 8 \text{ years}) = \text{Rs.600,000}$$

$$\text{FVIF}(r, 8 \text{ years}) = \frac{\text{Rs.600,000}}{\text{Rs.200,000}} = 3.000$$

From the tables we find that

$$\text{FVIF}(15\%, 8 \text{ years}) = 3.059$$

This means that the implied interest rate is nearly 15%.

I would choose Rs.600,000 after 8 years from now because I find a return of 15% quite attractive.

27. Ravikiran deposits Rs.500,000 in a bank now. The interest rate is 9 percent and compounding is done quarterly. What will the deposit grow to after 5 years? If the inflation rate is 3 percent per year, what will be the value of the deposit after 5 years in terms of the current rupee?

**Solution:**

$$\begin{aligned}
FV_5 &= \text{Rs.}500,000 [1 + (0.09 / 4)]^{5 \times 4} \\
&= \text{Rs.}500,000 (1.0225)^{20} \\
&= \text{Rs.}500,000 \times 2.653 \\
&= \text{Rs.}780,255
\end{aligned}$$

If the inflation rate is 3 % per year, the value of Rs.780,255 5 years from now, in terms of the current rupees is:

$$\begin{aligned}
&\text{Rs.}780,255 \times \text{PVIF} (3\%, 5 \text{ years}) \\
= &\text{Rs.}780,255 \times 0.863 \\
= &\text{Rs.}673,360
\end{aligned}$$

28. A person requires Rs.100,000 at the beginning of each year from 2015 to 2019. Towards this, how much should he deposit (in equal amounts) at the end of each year from 2007 to 2011, if the interest rate is 10 percent.

**Solution:**

The discounted value of Rs.100,000 receivable at the beginning of each year from 2015 to 2019, evaluated as at the beginning of 2014 (or end of 2013) is:

$$\begin{aligned}
&\text{Rs.}100,000 \times \text{PVIFA} (10\%, 5 \text{ years}) \\
= &\text{Rs.}100,000 \times 3.791 \\
= &\text{Rs.}379,100
\end{aligned}$$

The discounted value of Rs.379,100 evaluated at the end of 2011 is

$$\begin{aligned}
&\text{Rs.}379,100 \times \text{PVIF} (10\%, 2 \text{ years}) \\
= &\text{Rs.}379,100 \times 0.826 \\
= &\text{Rs.}313,137
\end{aligned}$$

If A is the amount deposited at the end of each year from 2007 to 2011 then

$$A \times \text{FVIFA} (10\%, 5 \text{ years}) = \text{Rs.}313,137$$

$$A \times 6.105 = \text{Rs.}313,137$$

$$A = \text{Rs.}313,137 / 6.105$$

$$= \text{Rs.}51,292$$

29. You require Rs.250,000 at the beginning of each year from 2010 to 2012. How much should you deposit (in equal amounts) at the beginning of each year in 2007 and 2008? The interest rate is 8 percent.

**Solution:**

The discounted value of Rs.250,000 receivable at the beginning of each year from 2010 to 2012, evaluated as at the beginning of 2009 (or end of 2008) is:

$$\begin{aligned} & \text{Rs.}250,000 \times \text{PVIFA} (8 \%, 3 \text{ years}) \\ = & \text{Rs.}250,000 \times 2.577 = \text{Rs.}644,250 \end{aligned}$$

To have Rs. 644,250 at the end of 2008, let A be the amount that needs to be deposited at the beginning of 2007 and 2008. We then have

$$A \times (1+0.08) \times \text{FVIFA} (8\%, 2\text{years}) = 644,250$$

$$A \times 1.08 \times 2.080 = 644,250 \text{ or } A = 286,792$$

30. What is the present value of Rs.120,000 receivable annually for 20 years if the first receipt occurs after 8 years and the discount rate is 12 percent.

**Solution:**

The discounted value of the annuity of Rs.120,000 receivable for 20 years, evaluated as at the end of 7<sup>th</sup> year is:

$$\text{Rs.}120,000 \times \text{PVIFA} (12\%, 20 \text{ years}) = \text{Rs.}120,000 \times 7.469 = \text{Rs.}896,290$$

The present value of Rs. 896,290 is:

$$\begin{aligned} & \text{Rs.} 896,290 \times \text{PVIF} (12\%, 7 \text{ years}) \\ = & \text{Rs.} 896,290 \times 0.452 \\ = & \text{Rs.}405,119 \end{aligned}$$

31. What is the present value of Rs.89,760 receivable annually for 10 years if the first receipt occurs after 5 years and the discount rate is 9 percent.

**Solution:**

The discounted value of the annuity of Rs.89,760 receivable for 10 years, evaluated as at the end of 4<sup>th</sup> year is:

$$\text{Rs.} 89,760 \times \text{PVIFA} (9\%, 10 \text{ years}) = \text{Rs.} 89,760 \times 6.418 = \text{Rs.}576,080$$

The present value of Rs. 576,080 is:

$$\text{Rs.} 576,080 \times \text{PVIF} (9\%, 4 \text{ years})$$

$$= \text{Rs. } 576,080 \times 0.708$$

$$= \text{Rs. } 407,865$$

32. After eight years Mr. Tiwari will receive a pension of Rs.10,000 per month for 20 years. How much can Mr. Tiwari borrow now at 12 percent interest so that the borrowed amount can be paid with 40 percent of the pension amount? The interest will be accumulated till the first pension amount becomes receivable.

**Solution:**

Assuming that the monthly interest rate corresponding to an annual interest rate of 12% is 1%, the discounted value of an annuity of Rs.4,000 receivable at the end of each month for 240 months (20 years) is:

$$\begin{aligned} & \text{Rs. } 4,000 \times \text{PVIFA} (1\%, 240) \\ & \text{Rs. } 4,000 \times \frac{(1.01)^{240} - 1}{.01 (1.01)^{240}} = \text{Rs. } 363,278 \end{aligned}$$

If Mr. Tiwari borrows Rs. $P$  today on which the monthly interest rate is 1%

$$\begin{aligned} P \times (1.01)^{96} &= \text{Rs. } 363,278 \\ P \times 2.60 &= \text{Rs. } 363,278 \\ P &= \frac{\text{Rs. } 363,278}{2.60} = \text{Rs. } 139,722 \end{aligned}$$

33. After one year Mr. Khanna will receive a pension of Rs.15,000 per month for 30 years. How much can Mr. Khanna borrow now at 12 percent interest so that the borrowed amount can be paid with 25 percent of the pension amount? The interest will be accumulated till the first pension amount becomes receivable.

**Solution:**

25 per cent of the pension amount is

$$0.25 \times \text{Rs. } 15,000 = \text{Rs. } 3,750$$

Assuming that the monthly interest rate corresponding to an annual interest rate of 12% is 1%, the discounted value of an annuity of Rs.3,750 receivable at the end of each month for 360 months (30 years) is:

$$\begin{aligned} & \text{Rs. } 3,750 \times \text{PVIFA} (1\%, 360) \\ & \quad (1.01)^{360} - 1 \end{aligned}$$

$$\text{Rs.}3,750 \times \frac{\quad}{.01 (1.01)^{360}} = \text{Rs.}364,569$$

If Mr. Khanna borrows Rs.P today on which the monthly interest rate is 1%

$$P \times (1.01)^{12} = \text{Rs. } 364,569$$

$$P \times 1.127 = \text{Rs. } 364,569$$

$$P = \frac{\text{Rs. } 364,569}{1.127} = \text{Rs.}323,486$$

34. You buy a car with a bank loan of Rs.525,000. An instalment of Rs.25,000 is payable to the bank for each of 30 months towards the repayment of loan with interest. What interest rate does the bank charge?

**Solution:**

$$\text{Rs.}25,000 \times \text{PVIFA}(r, 30 \text{ months}) = \text{Rs.}525,000$$

$$\text{PVIFA}(r, 30 \text{ months}) = \text{Rs.}525,000 / \text{Rs.}25,000 = 21$$

From the tables we find that:

$$\text{PVIFA}(3\%, 30) = 19.600$$

$$\text{PVIFA}(2\%, 30) = 22.397$$

Using a linear interpolation

$$r = 2\% + \left( \frac{22.397 - 21.000}{22.397 - 19.600} \right) \times 1\%$$

$$= 2.50\%$$

Thus, the bank charges an interest rate of 2.50 % per month.

The corresponding effective rate of interest per annum is

$$[(1.0250)^{12} - 1] \times 100 = 34.49 \%$$

35. You take a bank loan of Rs.174,000 repayable with interest in 18 monthly instalments of Rs.12,000. What is the effective annual interest rate charged by the bank ?

**Solution:**

$$\text{Rs.}12,000 \times \text{PVIFA}(r, 18 \text{ months}) = \text{Rs.}174,000$$

$$\text{PVIFA}(r, 18 \text{ months}) = \text{Rs.}174,000 / \text{Rs.}12,000 = 14.5$$

From the tables we find that:

$$\text{PVIFA (2\%, 18)} = 14.992$$

$$\text{PVIFA (3\%, 18)} = 13.754$$

Using a linear interpolation

$$r = 2\% + \left( \frac{14.992 - 14.500}{14.992 - 13.754} \right) \times 1\%$$
$$= 2.397\%$$

Thus, the bank charges an interest rate of 2.397 % per month.

The corresponding effective rate of interest per annum is

$$[(1.02397)^{12} - 1] \times 100 = 32.88 \%$$

36. Metro Corporation has to retire Rs.20 million of debentures each at the end of 6, 7, and 8 years from now. How much should the firm deposit in a sinking fund account annually for 5 years, in order to meet the debenture retirement need? The net interest rate earned is 10 percent.

**Solution:**

The discounted value of the debentures to be redeemed between 6 to 8 years evaluated at the end of the 5<sup>th</sup> year is:

$$\text{Rs.20 million} \times \text{PVIFA (10\%, 3 years)}$$

$$= \text{Rs.20 million} \times 2.487$$

$$= \text{Rs.49.74million}$$

If A is the annual deposit to be made in the sinking fund for the years 1 to 5, then

$$A \times \text{FVIFA (10\%, 5 years)} = \text{Rs.49.74 million}$$

$$A \times 6.105 = \text{Rs.49.74 million}$$

$$A = \text{Rs.8,147,420}$$

37. Ankit Limited has to retire Rs.30 million of debentures each at the end of 7, 8, 9 and 10 years from now. How much should the firm deposit in a sinking fund account annually for 5 years, in order to meet the debenture retirement need? The net interest rate earned is 12 percent.

**Solution:**

The discounted value of the debentures to be redeemed between 7 to 10 years evaluated at the end of the 6<sup>th</sup> year is:

$$\text{Rs.30 million} \times \text{PVIFA} (12\%, 4 \text{ years})$$

$$= \text{Rs.30 million} \times 3.037$$

$$= \text{Rs.91.11 million}$$

If A is the annual deposit to be made in the sinking fund for the years 1 to 6, then

$$A \times \text{FVIFA} (12\%, 6 \text{ years}) = \text{Rs.91.11 million}$$

$$A \times 8.115 = \text{Rs. 91.11 million}$$

$$A = \text{Rs.11,227,357}$$

38. Mr. Mehta receives a provident fund amount of Rs.800,000. He deposits it in a bank which pays 9 percent interest. If he plans to withdraw Rs.100,000 at the end of each year, how long can he do so ?

**Solution:**

Let 'n' be the number of years for which a sum of Rs.100,000 can be withdrawn annually.

$$\text{Rs.100,000} \times \text{PVIFA} (9\%, n) = \text{Rs.800,000}$$

$$\text{PVIFA} (9\%, n) = \text{Rs.800,000} / \text{Rs.100,000} = 8.000$$

From the tables we find that

$$\text{PVIFA} (9\%, 14 \text{ years}) = 7.786$$

$$\text{PVIFA} (9\%, 15 \text{ years}) = 8.060$$

Using a linear interpolation we get

$$n = 14 + \left( \frac{8.000 - 7.786}{8.060 - 7.786} \right) \times 1 = 14.78 \text{ years}$$

39. Mr. Naresh wants to invest an amount of Rs. 400,000, in a finance company at an interest rate of 12 percent, with instructions to the company that the amount with interest be repaid to his son in equal instalments of Rs.100,000, for his education expenses . How long will his son get the amount?

**Solution:**

Let 'n' be the number of years for which a sum of Rs.100,000 can be withdrawn annually.

$$\text{Rs.100,000} \times \text{PVIFA} (12\%, n) = \text{Rs.400,000}$$



$$PVIFA(12\%, n) = \text{Rs.}400,000 / \text{Rs.}100,000 = 4$$

From the tables we find that

$$PVIFA(12\%, 5 \text{ years}) = 3.605$$

$$PVIFA(12\%, 6 \text{ years}) = 4.111$$

Using a linear interpolation we get

$$n = 5 + \left( \frac{4.000 - 3.605}{4.111 - 3.605} \right) \times 1 = 5.78 \text{ years}$$

40. Your company is taking a loan of 1,000,000, carrying an interest rate of 15 percent. The loan will be amortised in five equal instalments. What fraction of the instalment at the end of second year will represent principal repayment ?

**Solution:**

$$\text{Annual instalment} = \frac{1,000,000}{3.352} = 298,329$$

**Loan Amortisation Schedule**

<i>Year</i>	<i>Beg.</i>	<i>Instalment</i>	<i>Interest</i>	<i>Principal repayment</i>	<i>Balance</i>
1	1,000,000	298,329	150,000	148,329	851,671
2	851,671	298,329	127,751	170,578	681,093
			$170,578 / 298,329 = 0.572 \text{ or } 57.2\%$		

41. Anurag Limited borrows Rs.2,000,000 at an interest rate of 12 percent. The loan is to be repaid in 5 equal annual instalments payable at the end of each of the next 5 years. Prepare the loan amortisation schedule.

**Solution:**

$$\begin{aligned} \text{Equated annual installment} &= 2,000,000 / PVIFA(12\%,5) \\ &= 2,000,000 / 3.605 \\ &= \text{Rs.}554,785 \end{aligned}$$

***Loan Amortisation Schedule***

<i>Year</i>	<i>Beginning amount</i>	<i>Annual instalment</i>	<i>Interest</i>	<i>Principal repaid</i>	<i>Remaining balance</i>
1	2,000,000	554,785	240,000	314,785	1,685,215

2	1,685,215	554,785	202,226	352,559	1,332,656
3	1,332,656	554,785	159,919	394,866	937,790
4	937,790	554,785	112,535	442,250	495,540
5	495,540	554,785	59,465	495,320	220*

(\*) rounding off error

42. You want to borrow Rs.3,000,000 to buy a flat. You approach a housing company which charges 10 percent interest. You can pay Rs.400,000 per year toward loan amortisation. What should be the maturity period of the loan?

**Solution:**

Let  $n$  be the maturity period of the loan. The value of  $n$  can be obtained from the equation.

$$\begin{aligned} 400,000 \times \text{PVIFA}(10\%, n) &= 3,000,000 \\ \text{PVIFA}(10\%, n) &= 7.5 \end{aligned}$$

From the tables we find that

$$\begin{aligned} \text{PVIFA}(10\%, 14 \text{ years}) &= 7.367 \\ \text{PVIFA}(10\%, 15 \text{ years}) &= 7.606 \end{aligned}$$

Using a linear interpolation we get

$$n = 14 + \left( \frac{7.500 - 7.367}{7.606 - 7.367} \right) \times 1 = 14.56 \text{ years}$$

43. You want to borrow Rs.5,000,000 to buy a flat. You approach a housing company which charges 11 percent interest. You can pay Rs.600,000 per year toward loan amortisation. What should be the maturity period of the loan?

**Solution:**

Let  $n$  be the maturity period of the loan. The value of  $n$  can be obtained from the equation.

$$\begin{aligned} 600,000 \times \text{PVIFA}(11\%, n) &= 5,000,000 \\ \text{PVIFA}(11\%, n) &= 8.333 \end{aligned}$$

From the tables we find that

$$\begin{aligned} \text{PVIFA}(11\%, 20 \text{ years}) &= 7.963 \\ \text{PVIFA}(11\%, 25 \text{ years}) &= 8.422 \end{aligned}$$

Using linear interpolation we get

$$n = 20 + \left( \frac{8.333 - 7.963}{8.422 - 7.963} \right) \times 5 = 24.03 \text{ years}$$

44. You are negotiating with the government the right to mine 160,000 tons of iron ore per year for 20 years. The current price per ton of iron ore is Rs.3500 and it is expected to increase at the rate of 8 percent per year. What is the present value of the iron ore that you can mine if the discount rate is 15 percent

**Solution:**

$$\begin{aligned} \text{Expected value of iron ore mined during year 1} &= 160,000 \times 3500 \times 1.08 \\ &= \text{Rs.}604.8 \text{ million} \end{aligned}$$

Expected present value of the iron ore that can be mined over the next 20 years assuming a price escalation of 8% per annum in the price per ton of iron

$$= \text{Rs.} 604.8 \text{ million} \times \left( \frac{1 - (1 + g)^n / (1 + i)^n}{i - g} \right)$$

$$= \text{Rs.} 604.8 \text{ million} \times \left( \frac{1 - (1.08)^{20} / (1.15)^{20}}{0.15 - 0.08} \right)$$

$$= \text{Rs.} 604.8 \text{ million} \times 10.2173$$

$$= \text{Rs.} 6,179,423,040$$

45. You are negotiating with the government the right to mine 300,000 tons of iron ore per year for 25 years. The current price per ton of iron ore is Rs.3200 and it is expected to increase at the rate of 7 percent per year. What is the present value of the iron ore that you can mine if the discount rate is 18 percent

**Solution:**

$$\begin{aligned} \text{Expected value of iron ore mined during year 1} &= 300,000 \times 3200 \times 1.07 \\ &= \text{Rs.}1027.2 \text{ million} \end{aligned}$$

Expected present value of the iron ore that can be mined over the next 25 years assuming a price escalation of 7% per annum in the price per ton of iron

$$= \text{Rs.} 1027.2 \text{ million} \times \left( \frac{1 - (1 + g)^n / (1 + i)^n}{i - g} \right)$$

$$= \text{Rs.} 1027.2 \text{ million} \times \left( \frac{1 - (1.07)^{25} / (1.18)^{25}}{0.18 - 0.07} \right)$$

$$= \text{Rs. } 1027.2 \text{ million} \times 8.3036$$

$$= \text{Rs. } 8,529,457,920$$

46. As a winner of a competition, you can choose one of the following prizes:
- Rs. 800,000 now
  - Rs. 2,000,000 at the end of 8 years
  - Rs. 100,000 a year forever
  - Rs. 130,000 per year for 12 years
  - Rs. 32,000 next year and rising thereafter by 8 percent per year forever.

If the interest rate is 12 percent, which prize has the highest present value?

**Solution:**

- $PV = \text{Rs. } 800,000$
- $PV = 2,000,000 \text{PVIF}_{12\%, 8\text{yrs}} = 2,000,000 \times 0.404 = \text{Rs. } 808,000$
- $PV = 100,000/r = 100,000/0.12 = \text{Rs. } 833,333$
- $PV = 130,000 \text{PVIFA}_{12\%, 12\text{yrs}} = 130,000 \times 6.194 = \text{Rs. } 805,220$
- $PV = C/(r-g) = 32,000/(0.12-0.08) = \text{Rs. } 800,000$

Option c has the highest present value viz. Rs.833,333

47. Oil India owns an oil pipeline which will generate Rs. 20 million of cash income in the coming year. It has a very long life with virtually negligible operating costs. The volume of oil shipped, however, will decline over time and, hence, cash flows will decrease by 4 percent per year. The discount rate is 15 percent.
- If the pipeline is used forever, what is the present value of its cash flows?
  - If the pipeline is scrapped after 30 years, what is the present value of its cash flows?

**Solution:**

$$(a) \quad PV = c/(r - g) = 20/[0.15 - (-0.04)] = \text{Rs. } 105.26 \text{ million}$$

$$(b) \quad PV = A(1+g) \left( \frac{1 - \left( \frac{1+g}{1+r} \right)^n}{r - g} \right) = 20 \times 0.96 \times 5.2398 = \text{Rs. } 100.604 \text{ million}$$

48. Petrolite owns an oil pipeline which will generate Rs. 15 million of cash income in the coming year. It has a very long life with virtually negligible operating costs. The volume of oil shipped, however, will decline over time and, hence, cash flows will decrease by 6 percent per year. The discount rate is 18 percent.
- If the pipeline is used forever, what is the present value of its cash flows?

- b. If the pipeline is scrapped after 10 years, what is the present value of its cash flows?

**Solution:**

(a)  $PV = c/(r - g) = 15/[0.18 - (-0.06)] = \text{Rs.}62.5 \text{ million}$

(b) 
$$PV = A(1+g) \left( \frac{1 - \left( \frac{1+g}{1+r} \right)^n}{\dots} \right) = 15 \times 0.94 \times 3.7379 = \text{Rs.}52.704 \text{ million}$$