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...CHY'S INTEGRAL THEOREM

→ If a function $f(z)$ is analytic and its derivative $f'(z)$ continous at all points inside and on a simple closed curve C then

$$\int_C f(z)\,dz = 0$$

Proof → Let the region enclosed by the curve c be R and let

$f(z) = u + iv$, $z = x + iy$ so $dz = dx + idy$

$$= \int_C f(z)\,dz = \int_C (u+iv)(dx+idy)$$

$$= \int_C (u\,dx + iu\,dy + iv\,dx - v\,dy)$$

Since $f'(z)$ is continous the partial derivatives $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ are also continuous in R. Hence By Green's Theorem we have

$$= \int_C (u\,dx - v\,dy) + i(v\,dx + u\,dy)$$

By Green's Theorem

$$= \iint_R \left(-\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dx\,dy + i\iint_R \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)$$

Green's Theorem → If $\phi(x,y)$, $\psi(x,y)$, $\frac{\partial \phi}{\partial y}$ and $\frac{\partial \psi}{\partial x}$ continous function over a region R bounded by simple closed curve C in x-y plane, then

$$\oint_C \phi\,dx + \psi\,dy = \iint_R \left(\frac{\partial \psi}{\partial x} - \frac{\partial \phi}{\partial y}\right) dx\,dy$$

Now $f(z)$ being analytic at each point of the region R, By Cauchy-Riema...

Replacing

$$-\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$

$$\& \ \frac{\partial v}{\partial y} = \frac{\partial u}{\partial x}$$

$$= \iint_R \left(\frac{\partial u}{\partial y} - \frac{\partial u}{\partial y}\right) dx\,dy + i\iint_R \left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial x}\right)$$

$$= 0 + i0$$

$$\int_C f(z)\,dz = 0$$

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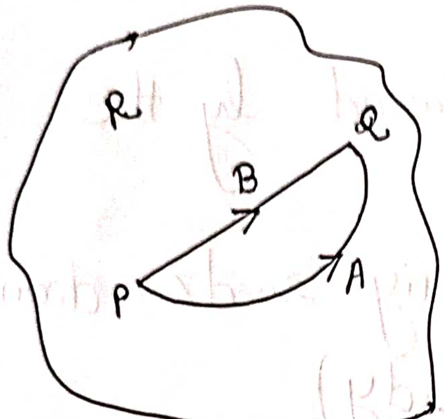
Corollary → If $f(z)$ is analytic in region R and P, Q are two points in then $\int_P^Q f(z)\,dz$ is indebendent of the path j... P and Q and lying entirely in R



let PAQ and PBQ be any two paths joining P and Q

By Cauchy's Theorem

$$\int_{PAQBP} f(z)\,dz = 0$$

$$\Rightarrow \int_{PAQ} f(z)\,dz + \int_{QBP} f(z)\,dz = 0$$

$$\Rightarrow \int_{PAQ} f(z)\,dz - \int_{PBQ} f(z)\,dz = 0$$

Hence $\displaystyle\int_{PAQ} f(z)\,dz = \int_{PBQ} f(z)\,dz$