विश्वविद्यालय

COMPLEX INTEGRATION

COMPLEX INTEGRATION → In case of real variable, the path of integration of $\int_a^b f(x)\,dx$ is always along the X-axis from $x = a$ to $x = b$.

But in case of a complex ~~variable~~ function $f(z)$ the path of the definite integral $\int_a^b f(z)\,dz$ can be along any curve from $z = a$ to $z = b$. Its value depends upon the path (curve) of integration. But the value of integral from a to b remains the same if the different curves from a to b are regular curves.

Example → Find the value of the integral

$$\int_C (x+y)\,dx + x^2y\,dy$$

(a) along $y = x^2$ having $(0,0)$, $(3,9)$ end points

(b) along $y = 3x$ between $(0,0)$ and $(3,9)$

Do the value depends upon path

Solution → $\int_C (x+y)\,dx + x^2y\,dy$

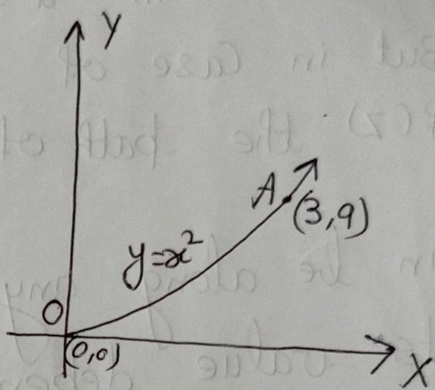
$P = x + y$ $\qquad Q = x^2y$ $\qquad \frac{\partial P}{\partial y} = 1$ $\quad \frac{\partial Q}{\partial x} = 2xy$

$\frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}$

The Integrals are not independent of path

(a) along $y = x^2$

$dy = 2x\,dx$



$$\int_0^3 (x+x^2)\,dx + x^2 \cdot x^2 \cdot 2x\,dx$$

$$= \int_0^3 \left(x + x^2 + 2x^5\right) dx$$

$$= \left[\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^6}{3}\right]_0^3$$

$$= \frac{9}{2} + 9 + 243 = 256\frac{1}{2}$$

(b) along $y = 3x$ $dy = 3\,dx$

$$= \int_0^3 (x+3x)\,dx + x^2 \cdot 3x \cdot (3\,dx$$

$$= \int_0^3 (4x + 9x^3)\,dx$$

$$= \left[2x^2 + \frac{9x^4}{4}\right]_0^3$$

$$= 18 + \frac{729}{4}$$

$$= 200\frac{1}{4}$$