SHOCK WAVE IS COMPRESSIBLE IN NATURE

from the Prandtl's relation

$$u_1 u_2 = a_*^2 \rightarrow (1)$$

from Equation (1) we have the Cases

(1) If $u_1 > a_*$ then $u_2 < a_*$

(2) If $u_1 < a_*$ then $u_2 > a_*$

It is necessary to decide which one of these alternative is valid.

It is necessary to use the fact that there must be entropy gain across the shock wave.

If s be the specific entropy then

$$\frac{s - s_0}{c_v} = \log \frac{p}{\rho^\gamma}$$

let s_1, s_2 be the specific entropies per unit mass on each side of the shock

So in region (1) & (2) we have

$$\frac{s_1 - s_0}{c_v} = \log \frac{p_1}{\rho_1^\gamma}$$

$$\frac{s_2 - s_0}{c_v} = \log \frac{p_2}{\rho_2^\gamma}$$

$$\frac{S_2 - S_0}{c_v} - \frac{S_1 - S_0}{c_v} = \log \frac{p_2}{\rho_2^{\gamma}} - \log \frac{p_1}{\rho_1^{\gamma}}$$

$$\frac{S_2 - S_1}{c_v} = \log\left[\frac{p_2}{\rho_2^{\gamma}} \times \frac{\rho_1^{\gamma}}{p_1}\right]$$

$$= \log\left[\frac{p_2}{p_1}\left(\frac{\rho_1}{\rho_2}\right)^{\gamma}\right]$$

using Eq^n (14) & (15)

$$= \log\left[\frac{2\gamma M_1^2 - (\gamma-1)}{\gamma+1} \times \left\{\frac{(\gamma-1)M_1^2 + 2}{(\gamma+1)M_1^2}\right\}^{\gamma}\right]$$

let $M_1^2 = 1+m$ where m is small

$$\frac{S_2 - S_1}{c_v} = \log\left[\frac{2\gamma(1+m) - (\gamma-1)}{\gamma+1} \times \left\{\frac{2 + (\gamma-1)(1+m)}{(\gamma+1)(1+m)}\right\}^{\gamma}\right]$$

$$= \log\left[\frac{2\gamma + 2\gamma m - \gamma + 1}{\gamma+1} \times \left\{\frac{2 + (\gamma-1)(1+m)}{(\gamma+1)(1+m)}\right\}^{\gamma}\right]$$

$$= \log\left[\frac{(\gamma+1) + 2\gamma m}{\gamma+1} \times \left\{\frac{2 + (\gamma-1) + m(\gamma-1)}{(\gamma+1)(1+m)}\right\}^{\gamma}\right]$$

$$= \log\left[\left(1 + \frac{2\gamma m}{\gamma+1}\right) \times \left\{\frac{(\gamma+1) + m(\gamma-1)}{\gamma+1}\right\}^{\gamma} (1+m)^{-\gamma}\right]$$

$$= \log\left[\left(1 + \frac{2\gamma m}{\gamma+1}\right)\left\{1 + \frac{m(\gamma-1)}{\gamma+1}\right\}^{\gamma} (1+m)^{-\gamma}\right]$$

$$= \log\left(1 + \frac{2\gamma m}{\gamma+1}\right) + \gamma \log\left(1 + \frac{\gamma-1}{\gamma+1} m\right) - \gamma \log(1+m)$$

$$\frac{S_2-S_1}{c_V} = \frac{2}{3}\gamma(\gamma-1)(\gamma+1)^{-2} m^3$$

$$+ - - -$$

Since for the shock wave $\frac{S_2-S_1}{c_V} > 0$ and $\gamma > 1$

this implies $m > 0$ and hence $M_1^2 = 1+m > 1$

$$\Rightarrow M_1^2 > 1$$

So

$$\left(\frac{u_1}{a_*}\right)^2 = \frac{\gamma+1}{\gamma-1+\frac{2}{M_1^2}} > \frac{\gamma+1}{(\gamma-1)+2} = 1$$

or $u_1 > a_*$ and so $u_2 < a_*$

This implies $\left(\frac{u_1}{a_*}\right)^2 > 1 \Rightarrow u_1 > a_*$ and $u_2 <$

Thus Case (i) holds & Case (ii) is false

Similarly it can be shown that $M_2 < 1$

In other words we have shown that for normal stationary shock wave, the up-st flow is supersonic but passage through th shock reduces it to subsonic

for $M_1 > 1$ we have show that $\frac{p_2}{p_1} = \frac{2\gamma M_1^2 - \gamma-1}{\gamma+1}$

$$\Rightarrow p_2 > p_1$$