19	20	21	22	23	24	25	26	27	28	29	30	31	कुल दिवस	आकस्मिक अवकाश की तिथियाँ	विशेष विवरण

Example If $f(z) = u(r,\theta) + iv(r,\theta)$ be an analytic function if $u = -r^3 \sin 3\theta$ then construct the corresponding analytic function $f(z)$ in terms of z

Solution

$$u = -r^3 \sin 3\theta$$

$$\frac{\partial u}{\partial r} = -3r^2 \sin 3\theta \qquad \frac{\partial u}{\partial \theta} = -3r^3 \cos 3\theta$$

$$dv = \frac{\partial v}{\partial r} dr + \frac{\partial v}{\partial \theta} d\theta$$

C-R Equations

$$= \left(-\frac{1}{r}\frac{\partial u}{\partial \theta}\right) dr + \left(r \frac{\partial u}{\partial r}\right) d\theta \qquad \frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$$

$$= -\frac{1}{r}(-3r^3 \cos 3\theta) dr + r(-3r^2 \sin 3\theta) d\theta \qquad \frac{\partial u}{\partial \theta} = -r\frac{\partial v}{\partial r}$$

$$dv = 3r^2 \cos 3\theta \, dr - 3r^3 \sin 3\theta \, d\theta$$

Here $\frac{\partial M}{\partial \theta} = \frac{\partial N}{\partial r}$

Exact

on Integration

$$\int dv = \int 3r^2 \cos 3\theta \, dr + \int \cancel{-3r^3 \sin 3\theta \, d\theta}$$

Terms do not involve r do

$$v = \int 3r^2 \cos 3\theta \, dr + C$$

$$r^3 \cos 3\theta + C$$

$$f(z) = u + iv = -r^3 \sin 3\theta + i r^3 \cos 3\theta + iC$$

$$= i r^3 (\cos 3\theta + i \sin 3\theta) + iC$$

$$= i r^3 e^{i3\theta} + iC$$

$$= +i (re^{i\theta})^3 + iC$$

$$= i z^3 + iC \quad \text{Ans.}$$

Example → Find analytic function $f(z) = u$ ...

such that $v(r, \theta) = r^2 \cos 2\theta - r\cos$ ...

Solution → $\frac{\partial v}{\partial \theta} = -2r^2 \sin 2\theta + r\sin\theta$

$\frac{\partial v}{\partial r} = 2r\cos 2\theta - \cos\theta$

using C-R Equations

$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$

$\frac{\partial u}{\partial \theta} = -r\frac{\partial v}{\partial r}$

$du = \frac{\partial u}{\partial r} dr + \frac{\partial u}{\partial \theta} d\theta$

$du = \left(\frac{1}{r}\frac{\partial v}{\partial \theta}\right) dr + \left(-r\frac{\partial v}{\partial r}\right) d\theta$

$= (-2r\sin 2\theta + \sin\theta)\, dr + (-2r^2\cos 2\theta + r\cos\theta)\, d\theta$

$= -\left[(2r\,dr)\{\sin 2\theta\} + r^2(\cos 2\theta)d\theta\right] + \left[\sin\theta . dr + r.(\cos\theta)d\theta\right]$

$= -d(r^2 \sin 2\theta) + d(r\sin\theta)$

or

$u = \int M\, dr + \int N$ (does not involve r) $d\theta$

$u = \int -2r\sin 2\theta + \sin\theta\, dr + 0 + c$

$u = -r^2 \sin 2\theta + r\sin\theta + c$

on integration

$u = -r^2 \sin 2\theta + r\sin\theta + C$

$f(z) = u + iv$

$= (-r^2 \sin 2\theta + r\sin\theta + c) + i(r^2\cos 2\theta - r\cos\theta + 2)$

$= ir^2[\cos 2\theta + i\sin 2\theta] - ir[\cos\theta + i\sin\theta] + 2i + C$

$= ir^2 e^{i2\theta} - ire^{i\theta} + 2i + C$

$= i[r^2 e^{i2\theta} - re^{i\theta}] + 2i + c$ Ans.

$= i[z^2 - z] + 2i + c$