METHOD TO FIND THE CONJUGATE FUNCTION

Given → If $f(z) = u + iv$, and u is known and $u(x,y)$ and $v(x,y)$ are conjugate functions.

To Find → v, Conjugate function

Method → We know $v = v(x,y)$

$$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy$$

using C-R Equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$dv = -\frac{\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy$$

$$v = \int -\frac{\partial u}{\partial y}dx + \int \frac{\partial u}{\partial x}\cdot dy \quad \boxed{\text{or}} \quad v = \int M dx + \int N dy \quad \text{(1)}$$

or $M = -\frac{\partial u}{\partial y}$ $\qquad N = \frac{\partial u}{\partial x}$

$$\frac{\partial M}{\partial y} = -\frac{\partial^2 u}{\partial y^2} \qquad \frac{\partial N}{\partial x} = \frac{\partial^2 u}{\partial x^2}$$

Since u is a conjugate function (given)

so $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

$$\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 0 \quad \text{or} \quad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \text{(2)}$$

Eqn (2) satisfies the condition of an Exact differential Equation

so Equation (1) can be integrated & thus v is determined.

छत्रपति शाहू जी महारा

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उपस्थिति

Problem → let $f(z) = u(x,y) + iv(x,y)$ be an analytic function. If $u = 3x - 2xy$ then find v and Express $f(z)$ in terms of z

Solution → $u = 3x - 2xy$

$$\frac{\partial u}{\partial x} = 3 - 2y \qquad \frac{\partial u}{\partial y} = -2x$$

$$dv = \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dx$$

$$= \left(-\frac{\partial u}{\partial y}\right) dx + \left(\frac{\partial u}{\partial x}\right) dy$$

$$dv = \underbrace{2x\,dx}_{M\,dx} + \underbrace{(3-2y)\,dy}_{N\,dy}$$

$\frac{\partial M}{\partial y} = 0$

$\frac{\partial N}{\partial x} = 0$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

So Eq^n is Exact

Now Solution is

$$v = \int 2x\,dx + \int (3-2y)\,dy$$

$$= x^2 + 3y - y^2 + C$$

$$f(z) = u(x,y) + iv(x,y)$$

$$= (3x - 2xy) + i(x^2 + 3y - y^2 + C)$$

$$= (ix^2 - iy^2 - 2xy) + (3x + i3y) + iC$$

$$= i(x^2 - y^2 + 2ixy) + 3(x+iy) + iC$$

$$= i(x+iy)^2 + 3(x+iy) + iC$$

$$= iz^2 + 3z + iC$$