Classification and formation of Differential Equations

Differential equations → Any relation between the dependent and independent variable is called differential equation if it involves differential coefficient(s) of the dependent variable with respect to the independent variable(s)

Classification of Differential Equation

Differential equations are classified as below.

(1) Ordinary Differential equations (O.D.E.) → A differential equation which involves only one independent variable is called an ordinary differential equation. For instance

$$\frac{dy}{dx} = m \qquad (1)$$

$$\frac{d^2y}{dx^2} + a^2y = \sec ax \qquad (2)$$

$$\frac{d^2y}{dx^2} + 2y\frac{dy}{dx} + \sqrt{1+\left(\frac{dy}{dx}\right)^2} = 0 \qquad (3)$$

are ordinary differential equations because each one of them involves only one independent variable x. Here y is dependent variable.

② Partial Differential Equations (P.D.E.)

A differential equation which involves the partial differential coefficients with respect to more than one independent variables is called a partial differential equation. For instance

$$\frac{\partial u}{\partial x} = k\frac{\partial u}{\partial y} \rightarrow (1)$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \rightarrow (2)$$

are partial differential equations. In both the equations u is the dependent variable. In (1) x and y are the independent variables whereas in (2) these variables are x, y and z.

Order of a Differential Equation → The order of a differential equation is the order of the highest differential coefficient which occurs in it.

$\frac{\partial z}{\partial x} = m$ and $\frac{\partial u}{\partial x} = k\frac{\partial u}{\partial y}$ are of the first order

$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \sin x$, $\frac{\partial^2 z}{\partial x^2} + $ [illegible]

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$ are of the second order

Degree of a Differential equation →

The degree of a differential Equation is the degree of the highest differential coefficient which occurs in it derivative after removing the radical sign and fraction

$\frac{dy}{dx} = m$

$\frac{d^2y}{dx^2} + a^2y = \sec ax$

$\frac{\partial u}{\partial x} = k \frac{\partial u}{\partial y}$

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$ are of the first degree

$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$ The degree of this Equation is 2. and order = 2

$\cos^2 x \frac{d^2y}{dx^2} + \sin x \left(\frac{dy}{dx}\right)^2 + 8y = \tan x$

order = 2

degree = 1