eigen vectors for the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

...on → We Know that the characteristic Equation of A is $|A - \lambda I| = 0$

$$\begin{vmatrix} 0-\lambda & 1 & 1 \\ 1 & 0-\lambda & -1 \\ 1 & -1 & 0-\lambda \end{vmatrix} = 0$$

$$-\lambda(\lambda^2 - 1) - 1(-\lambda + 1) + 1(-1 + \lambda) = 0$$

$$-\lambda(\lambda-1)(\lambda+1) + 1(\lambda-1) + 1(\lambda-1) = 0$$

$$(\lambda-1)[-\lambda(\lambda+1) + 2] = 0$$

$$(\lambda-1)[-\lambda^2 - \lambda - 2] = 0$$

$$(\lambda-1)(\lambda^2 + \lambda - 2] = 0$$

$$(\lambda-1)(\lambda-1)(\lambda+2) = 0$$

Hence $\lambda = 1, 1, -2$

The eigen vector corresponding to the root $\lambda = 1$ is given by

$$\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{l} \\ R_2 + R_1 \\ R_3 + R_1 \end{array} \begin{bmatrix} -1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

This given only one Equation

$$-x_1 + x_2 + x_3 = 0$$

The Solution of this Equation can be given as

$x_2 = k_1,$

$x_1 = k_1 + k_2,$

where k_1 and k_2 are scalers, not bo

Hence for $\lambda = 1$, eigen vector is given by

$$X = \begin{bmatrix} k_1 + k_2 \\ k_1 \\ k_2 \end{bmatrix}$$ where, k_1, k_2 are scalers not both zero

Further for $\lambda = -2$, the Eigen vector is given by

$$\cancel{\begin{bmatrix} 0 & 0 & 0 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}} \quad \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2x_1 + x_2 + x_3 = 0$$
$$x_1 + 2x_2 - x_3 = 0$$
$$x_1 - x_2 + 2x_3 = 0$$

$$\frac{x_1}{4-1} = \frac{-x_2}{2+1} = \frac{x_3}{-1-2} \cancel{= k_3 \text{ (say)}}$$

$$\frac{x_1}{3} = \frac{x_2}{-3} = \frac{x_3}{-3}$$

$$\frac{x_1}{1} = \frac{x_2}{-1} = \frac{x_3}{-1} = k_3 \text{ (say)}$$

Hence for $\lambda = -2$ eigen vector is given by

$$X = \begin{bmatrix} k_3 \\ -k_3 \\ -k_3 \end{bmatrix} = k_3 \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$ where k_3 is a n zero scaler