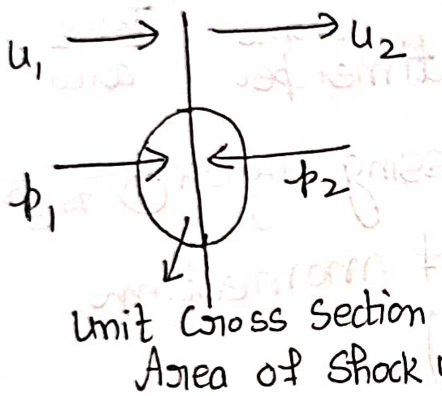
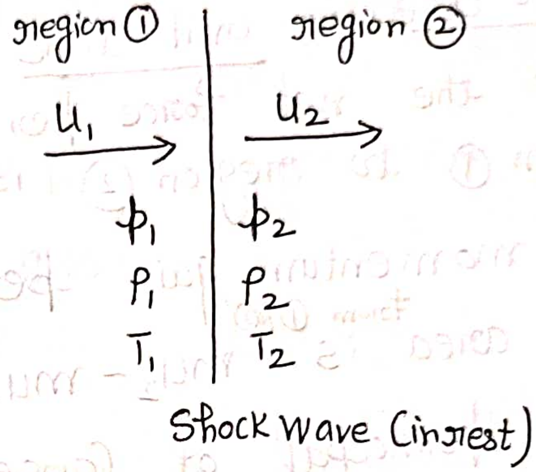
Elementary Analysis Of The Normal Shock Wave

Normal Shock Wave → If the fluid particle is moving perpendicular to the shock wave in both the region i.e. ahead and behind the shock wave, then it is called Normal Shock Wave.





Here we treat the case of stationary normal shock. If, as in the model of the piston advancing into the shock tube, the shock is not stationary, the problem can always be treated by reducing the shock to rest by imposing an equal & opposite velocity onto the entire system

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Figure ① shows that the normal shock wave in rest separating the two region ① & ② let the pressure, density, temp. and particle velocity in region ① and ② respectively p_1, ρ_1, T_1, u_1 and p_2, ρ_2, T_2, u_2.

Let the flow ~~particle~~ variables in region ① are known. We will find flow variable in region ② by continuity, momentum and energy considerations.

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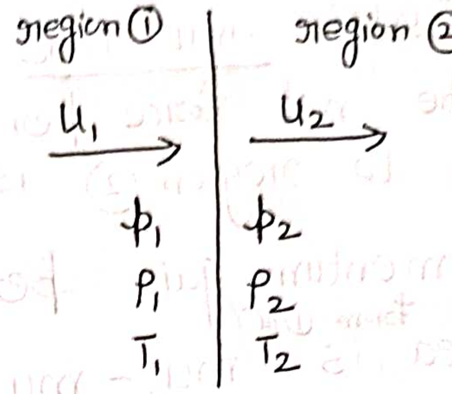
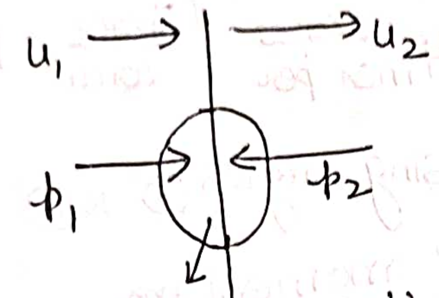


Shock wave (in rest)



Unit Cross Section Area of Shock wave

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densi $= \frac{m}{V}$

$m = \rho \times v$
$= \rho \times A \times l$
$= \rho \times A \times \frac{u}{t}$

If $A=1$, $t=1$, $m = \rho u$

By Continuity Consideration, we al[...]

$$\rho_1 u_1 = \rho_2 u_2 = m(\text{say}) \rightarrow ①$$

the mass [...] gas per [...] time per unit [...] crossing the wave front region ① is $\rho_1 u_1$. That entering Region ② is $\rho_2 u_2$

where m is the mass of the gas from region ① to region ② across the shock front through unit cross section area in unit time

Since the net force per unit area of shock from region ① to region ② is $p_1 - p_2$

and momentum gain per unit time per unit area, across from ① to ②, is $mu_2 - mu_1$ in crossing region ① to ②

$\frac{\text{force}}{\text{area}} = \text{pressure } (p)$

By principal of Conservation of momentum

$$p_1 - p_2 = m(u_2 - u_1)$$

$$\boxed{p_1 + mu_1 = p_2 + mu_2} \rightarrow ②$$

Now the rate at which the pressures do work on the mass flux m is $\boxed{p_1 u_1 - p_2 u_2}$

The energy of unit mass of the gas in the region ① is

$$= K.E + \text{~~Potential~~ Internal energy}$$

$$= \frac{1}{2}u_1^2 + C_V T_1$$

here m = 1

$C_V = \frac{R}{\gamma - 1}$

$$= \frac{1}{2}u_1^2 + \frac{R}{\gamma - 1} T_1$$

$PV = RT$

$\frac{P}{\rho} = RT$ $\quad V = \frac{1}{\rho}$

$\frac{p_1}{\rho_1} = RT_1$

$$= \frac{1}{2}u_1^2 + \frac{p_1}{\rho_1(\gamma - 1)}$$

Similary, Energy of unit mass of gas in region ② is

$$\frac{1}{2}u_2^2 + \frac{p_2}{(\gamma-1)\rho_2}$$

So Increase in the energy of unit mass of gas in crossing region ① to ② is

$$\left(\frac{1}{2}u_2^2 + \frac{p_2}{(\gamma-1)\rho_2}\right) - \left(\frac{1}{2}u_1^2 + \frac{p_1}{(\gamma-1)\rho_1}\right)$$

The Increase in energy in unit time across the unit area of front of mass m of gas in crossing region ① to ②

$$= m\left(\frac{1}{2}u_2^2 + \frac{p_2}{(\gamma-1)\rho_2}\right) - m\left(\frac{1}{2}u_1^2 + \frac{p_1}{(\gamma-1)\rho_1}\right)$$

Thus By Conservation of energy

$$p_1u_1 - p_2u_2 = m\left(\frac{1}{2}u_2^2 + \frac{p_2}{(\gamma-1)\rho_2}\right) - m\left(\frac{1}{2}u_1^2 + \frac{p_1}{(\gamma-1)\rho_1}\right)$$

$$\Rightarrow \left(\frac{1}{2}u_2^2 + \frac{p_2}{(\gamma-1)\rho_2}\right) - \left(\frac{1}{2}u_1^2 + \frac{p_1}{\rho_1(\gamma-1)}\right) = \frac{p_1u_1}{m} - \frac{p_2u_2}{m}$$

$$= \frac{p_1u_1}{\rho_1u_1} - \frac{p_2u_2}{\rho_2u_2} \qquad \boxed{\rho_1u_1 = \rho_2u_2 = m}$$

$$\left(\frac{1}{2}u_2^2 + \frac{p_2}{(\gamma-1)\rho_2}\right) - \left(\frac{1}{2}u_1^2 + \frac{p_1}{\rho_1(\gamma-1)}\right) = \frac{p_1}{\rho_1} - \frac{p_2}{\rho_2}$$

$$\frac{1}{2}u_2^2 + \frac{p_2}{\rho_2(\gamma-1)} + \frac{p_2}{\rho_2} = \frac{1}{2}u_1^2 + \frac{p_1}{(\gamma-1)\rho_1} + \frac{p_1}{\rho_1}$$

$$\frac{1}{2}u_2^2 + \frac{p_2}{\rho_2}\left[\frac{\cancel{1}+\gamma-\cancel{1}}{\gamma-1}\right] = \frac{1}{2}u_1^2 + \frac{p_1}{\rho_1}\left[\frac{1+\gamma-1}{\gamma-1}\right]$$