IONS REDUCIBLE TO THE LINEAR FORM ③

(BERNOULLI'S EQUATION)

Equation of the form

$$\frac{dy}{dx} + Py = Q.y^n \rightarrow ①$$

where P and Q are constants or functions of x can be reduced to the linear form on dividing by y^n and substituting $\frac{1}{y^{n-1}} = z$

on dividing both sides of ① by y^n we get

$$\frac{1}{y^n}\frac{dy}{dx} + \frac{1}{y^{n-1}}P = Q$$

Put $\frac{1}{y^{n-1}} = z$

or $y^{1-n} = z$

$(1-n)y^{1-n-1}\frac{dy}{dx} = \frac{dz}{dx}$

$(1-n)y^{-n}\frac{dy}{dx} = \frac{dz}{dx}$

$\frac{(1-n)}{y^n}\frac{dy}{dx} = \frac{dz}{dx}$

so that

$$\frac{1}{1-n}\frac{dz}{dx} + Pz = Q$$

or $$\frac{dz}{dx} + P(1-n)z = Q(1-n)$$

which is a linear equation and can be solved easily by previous method.

Solve the special Bernoulli Equation

$$y' - Ay = -By^2 \rightarrow ①$$ (A, B Positive constants)

Solution

$$\frac{dy}{dx} - Ay = -By^2$$

$$\frac{dy}{dx} + Py = Q \cdot y^n$$

here $n = 2$,

$$Z = \frac{1}{y^{2-1}} \qquad Z = \frac{1}{y}$$

$$\frac{dz}{dx} = -y^{-2}\frac{dy}{dx}$$

$$\frac{dz}{dx} = -\frac{1}{y^2}\frac{dy}{dx} \rightarrow \text{Put from } ①$$

$$\frac{dz}{dx} = -\frac{1}{y^2}\left[-By^2 + Ay\right]$$

$$\frac{dz}{dx} = B - \frac{A}{y} \qquad \text{here } \frac{1}{y} = z$$

$$\frac{dz}{dx} = B - Az$$

$$\frac{dz}{dx} + Az = B$$

linear form

$$\frac{dy}{dx} + Py = Q$$

$P = A$

$$I.F = e^{\int A\,dx} = e^{Ax}$$

general solution

$$z \cdot [I.F] = \int Q \cdot [I.F]\,dx + c$$

$$z \cdot e^{Ax} = \int B \cdot e^{Ax}\,dx + C$$

$$z = e^{-Ax}\left[\frac{e^{Ax}}{A} \cdot B + c\right]$$

$$z = ce^{-Ax} + \frac{B}{A}$$

Now the general solution of ① is

$$y = \frac{1}{z} = \frac{1}{\frac{B}{A} + ce^{-Ax}}$$

Answer.