

Equations reducible to homogeneous form \rightarrow

The Equation of the form

$$\frac{dy}{dx} = \frac{ax + by + c}{Ax + By + C}$$

can be reduced to the homogeneous form by the substitution

$$x = X + h, \quad y = Y + k$$

$$\frac{dy}{dx} = \frac{dY}{dX}$$

(h, k being constants)

The given differential Equation reduces to

$$\begin{aligned} \frac{dy}{dx} &= \frac{a(x+h) + b(y+k) + c}{A(x+h) + B(y+k) + C} \\ &= \frac{ax + by + ah + bk + c}{Ax + By + Ah + Bk + C} \end{aligned}$$

Choose h, k so that $ah + bk + c = 0$

$$Ah + Bk + c = 0$$

Then the given Equation become homogeneous

$$\frac{dY}{dX} = \frac{aX + bY}{AX + BY}$$

Case of failure \rightarrow If $\frac{a}{A} = \frac{b}{B}$ then the value of h, k will not be finite

$$\frac{a}{A} = \frac{b}{B} = \frac{1}{m} \text{ (say)}$$

$$A = am, \quad B = bm$$

The given Equation becomes $\frac{dy}{dx} = \frac{ax + by + c}{m(ax + by) + C}$
Now Put $ax + by = Z$ and apply the method of variable separable.

Handwritten musical notation on a page, featuring multiple staves with notes, clefs, and other musical symbols. The notation is dense and appears to be a complex piece of music, possibly a score for multiple instruments or voices. The page is oriented vertically, with the notation running from top to bottom. The handwriting is somewhat stylized and includes various musical notations such as notes, rests, and clefs. The page is numbered '11' in the top right corner.

11

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$$(1+u) du + \frac{3}{2(1-u)} du = \frac{dx}{x}$$

on integration

$$\frac{1}{2} \log(1+u) - \frac{3}{2} \log(1-u) = \log x + \log C$$

$$\log \frac{1+u}{(1-u)^3} = \log C^2 x^2$$

$$\frac{1+u}{(1-u)^3} = C^2 x^2$$

$$\text{Put } u = \frac{y}{x}$$

$$\frac{1 + \frac{y}{x}}{\left(1 - \frac{y}{x}\right)^3} = C^2 x^2$$

$$\frac{x+y}{(x-y)^3} = C^2$$

$$x+y = C^2 (x-y)^3$$

$$x-1 + y-1 = C^2 [x-y]^3$$

$$x+y-2 = C^2 (x-y)^3$$

Put
x =
y =