ations reducible to homogeneous form →

The Equation of the form

$$\frac{dy}{dx} = \frac{ax + by + c}{Ax + By + C}$$

can be reduced to the homogeneous form by the substitution

$$x = X + h, \quad y = Y + K \qquad (h, K \text{ being constants})$$

$$\frac{dy}{dx} = \frac{dY}{dX}$$

The given differential equation reduces to

$$\frac{dY}{dX} = \frac{a(X+h) + b(Y+K) + c}{A(X+h) + B(Y+K) + C}$$

$$= \frac{aX + bY + ah + bK + c}{AX + BY + Ah + BK + C}$$

Choose h, k so that $ah + bK + c = 0$

$$Ah + BK + C = 0$$

Then the given equation become homogeneous

$$\frac{dY}{dX} = \frac{aX + bY}{AX + BY}$$

Case of failure → If $\frac{a}{A} = \frac{b}{B}$ then the value of h, K will not be finite

$$\frac{a}{A} = \frac{b}{B} = \frac{1}{m} \text{ (say)}$$

$A = am, \quad B = bm$

The given Equation becomes $\frac{dy}{dx} = \frac{ax + by + c}{m(ax + by) + C}$

Now Put $ax + by = Z$ and apply the method of variable separable

Example - Solve $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$

Solution - Put $x = X+h$, $y = Y+k$

The given equation reduces to

$$\frac{dY}{dX} = \frac{(X+h)+2(Y+k)-3}{2(X+h)+(Y+k)-3}$$

$$= \frac{X+2Y+(h+2k-3)}{2X+Y+(2h+k-3)}$$

Now choose h and k so that $h+2k-3=0$

$2h+k-3=0$

on solving we get $h=1$, $k=1$

$$\frac{dY}{dX} = \frac{X+2Y}{2X+Y}$$

Put $Y = vX$

$$\frac{dY}{dX} = v + X\frac{dv}{dX}$$

$$v + X\frac{dv}{dX} = \frac{X+2vX}{2X+vX}$$

$$v + X\frac{dv}{dX} = \frac{1+2v}{2+v}$$

$$X\frac{dv}{dX} = \frac{1+2v}{2+v} - v = \frac{1-v^2}{2+v}$$

$$\left(\frac{2+v}{1-v^2}\right) dv = \frac{dX}{X}$$

$$\frac{2+v}{(1+v)(1-v)}\, dv = \frac{dX}{X}$$

$$\left[\frac{1}{2(1+v)} + \frac{3}{2(1-v)}\right] dv = \frac{dX}{X}$$ By Partial

$$\frac{1}{(1+v)}dv + \frac{3}{2(1-v)}dv = \frac{dx}{x}$$

on Integration

$$\frac{1}{2}\log(1+v) - \frac{3}{2}\log(1-v) = \log x + \log c$$

$$\log \frac{1+v}{(1-v)^3} = \log c^2 x^2$$

$$\frac{1+v}{(1-v)^3} = c^2 x^2$$

Put $v = \frac{y}{x}$

$$\frac{1+\frac{y}{x}}{\left(1-\frac{y}{x}\right)^3} = c^2 x^2$$

$$\frac{x+y}{(x-y)^3} = c^2$$

$$x+y = c^2(x-y)^3$$

Put

$x =$

$y =$

$$x-1+y-1 = c^2[x-y+1]^3$$

$$x+y-2 = c^2(x-y)^3$$