

INTEGRAL EQUATION

Introduction → The theory of Integral Equation is one of the most important branch of mathematical analysis. For the solution of differential and difference equation Laplace took the integral transform $\int_0^{\infty} e^{-xt} f(t) dt$.

Abel find the integral equation of the form $f(x) = \int_0^x (x-t)^{-\alpha} u(t) dt$ which is

Known as Abel's integral Equation.

Poisson also obtained an integral equation of the form

$$u(x) = f(x) + \int_0^x k(x,t) u(t) dt$$

in which the unknown function $u(t)$ occurs inside the integral sign.

Definition → An equation in which an unknown function appears under one or more integral signs is called an integral equation.

The name integral equation for any equation involving the unknown function $u(x)$ under the integral sign was introduced by Du Bois-Reymond.

In 1870, Neumann show that the Dirichlet problem in which the function y has the prescribed values over a certain boundary surface S and satisfied Laplace's Equation $\nabla^2 \psi = 0$ within the region enclosed by S is equivalent to the solution of an integral equation.

V. Volterra, gave the first general solution of the linear integral equation which is known as Volterra solution of integral equation.

I. Fredholm, gave another general solution of linear integral equation and solved the Dirichlet problem by this method which is in the form of

$$y(x) = f(x) + \lambda \int_a^b K(x,t) y(t) dt$$

which is known as Fredholm solution of an integral equation.