



# Introduction of K-Map

# Karnaugh Map(K-Map) method

The **K-map** is a systematic way of simplifying Boolean expressions. With the help of the K-map method, we can find the simplest POS and SOP expression, which is known as the minimum expression. The K-map provides a cookbook for simplification.

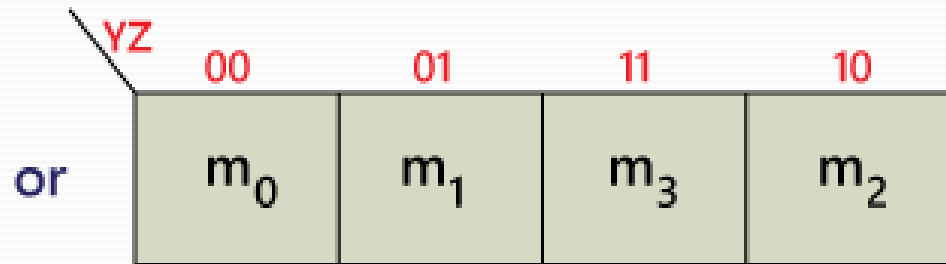
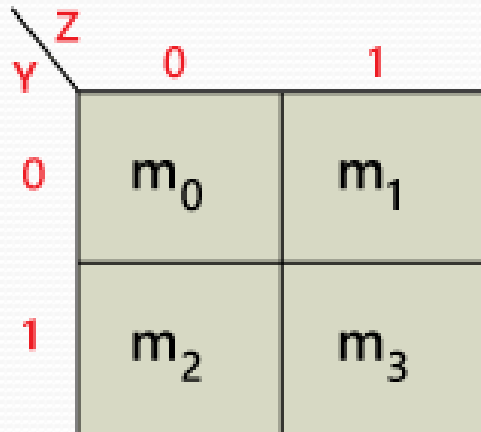
Just like the truth table, a K-map contains all the possible values of input variables and their corresponding output values. However, in K-map, the values are stored in cells of the array. In each cell, a binary value of each input variable is stored.

The K-map takes the SOP and POS forms. The K-map grid is filled using 0's and 1's. The K-map is solved by making groups. There are the following steps used to solve the expressions using K-map:

- First, we find the K-map as per the number of variables.
- Find the maxterm and minterm in the given expression.
- Fill cells of K-map for SOP with 1 respective to the minterms.
- Fill cells of the block for POS with 0 respective to the maxterm.
- Next, we create rectangular groups that contain total terms in the power of two like 2, 4, 8, ... and try to cover as many elements as we can in one group.
- With the help of these groups, we find the product terms and sum them up for the SOP form.

# 2 Variable K-map

There is a total of 4 variables in a 2-variable K-map. There are two variables in the 2-variable K-map. The following figure shows the structure of the 2-variable K-map:



# 3-variable K-map

- The 3-variable K-map is represented as an array of eight cells. In this case, we used A, B, and C for the variable. We can use any letter for the names of the variables. The binary values of variables A and B are along the left side, and the values of C are across the top.

		C	
		0	1
AB	00		
	01		
	11		
	10		

		C	
		0	1
AB	00	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$
	01	$\bar{A}B\bar{C}$	$\bar{A}BC$
	11	$AB\bar{C}$	$ABC$
	10	$A\bar{B}\bar{C}$	$A\bar{B}C$

# 4-Variable Karnaugh Map

- The 4-variable K-map is represented as an array of 16 cells. Binary values of A and B are along the left side, and the values of C and D are across the top. The value of the given cell is the binary values of A and B at left side in the same row combined with the binary values of C and D at the top in the same column.

AB \ CD	00	01	11	10
00				
01				
11				
10				

AB \ CD	00	01	11	10
00	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}\bar{B}C\bar{D}$	$\bar{A}\bar{B}CD$
01	$\bar{A}B\bar{C}\bar{D}$	$\bar{A}B\bar{C}D$	$\bar{A}BC\bar{D}$	$\bar{A}BCD$
11	$AB\bar{C}\bar{D}$	$AB\bar{C}D$	$ABC\bar{D}$	$ABCD$
10	$A\bar{B}\bar{C}\bar{D}$	$A\bar{B}\bar{C}D$	$A\bar{B}C\bar{D}$	$A\bar{B}CD$

# PROBLEMS BASED ON KARNAUGH MAP

- Minimize the following boolean function-
- $F(A, B, C, D) = \Sigma m(0, 1, 3, 5, 7, 8, 9, 11, 13, 15)$

## **Solution-**

- Since the given boolean expression has 4 variables, so we draw a 4 x 4 K Map.
- We fill the cells of K Map in accordance with the given boolean function.
- Then, we form the groups in accordance with the above rules.

$F(A, B, C, D)$

$$= (A'B' + A'B + AB + AB')(C'D + CD) + (A'B' + AB')(C'D' + C'D)$$

$$= D + B'C'$$

A 4x4 Karnaugh map for the function F(A, B, C, D). The columns are labeled with CD combinations:  $\bar{C}\bar{D}$ ,  $\bar{C}D$ ,  $CD$ , and  $C\bar{D}$ . The rows are labeled with AB combinations:  $\bar{A}\bar{B}$ ,  $\bar{A}B$ ,  $AB$ , and  $A\bar{B}$ . The map contains 1s in cells (0,0), (1,0), (2,0), (3,0), (1,1), (2,1), (1,2), (2,2), (0,3), and (3,3). Prime implicants are highlighted: a green box covers cells (0,0), (1,0), (0,3), and (3,3); a blue box covers cells (1,0), (1,1), (2,1), and (2,2); a vertical blue line covers cells (1,0), (1,1), (2,1), and (3,0); and a vertical green line covers cells (1,0) and (1,1).

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	$CD$	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	1	
$\bar{A}B$		1	1	
$AB$		1	1	
$A\bar{B}$	1	1	1	

Thus, minimized boolean expression is-

$$F(A, B, C, D) = B'C' + D$$